
DECISION MAKING IN COMPLEX ENVIRONMENTS

The Analytic Network Process (ANP) for Dependence and Feedback

**Including a Tutorial for the SuperDecisions Software
and
Portions of the Encyclicon of Applications, Vol. I**

Rozann W. Saaty



SUPER DECISIONS

Software for Decision Making with Dependence and Feedback

Copyrights and Acknowledgments

This material was assembled from works by Thomas L. Saaty and from student projects in his classes done over several years. The instructions on how to use the *SuperDecisions* software were prepared by Rozann W. Saaty of the Creative Decisions Foundation. The software that implements the Analytic Network Process, *SuperDecisions*, was developed by William J. Adams of Embry Riddle Aeronautical University, Daytona Beach, Florida, working with Rozann W. Saaty. The dictionary of ANP applications, the Encyclicon, included here as an appendix, was compiled from materials by Thomas L. Saaty and his students.

ISBN Number 1-888603-00-3

© Rozann W. Saaty 2016

This material may not be reproduced in any form without the permission of Rozann W. Saaty or Thomas L. Saaty.

Rozann W. Saaty
Creative Decisions Foundation
4922 Ellsworth Avenue
Pittsburgh, PA 15213
Phone: 412-621-6546
e-mail: rozann@creativdecisions.net

Thomas L. Saaty
322 Mervis Hall
Katz Graduate School of Business
University of Pittsburgh
Pittsburgh, PA 15260
Phone: 412-648-1539
e-mail: saaty@katz.pitt.edu

Biography

Thomas Saaty holds the Chair of University Professor at the University of Pittsburgh. He received his Ph.D. in mathematics with a minor in physics from Yale University. From 1963-1969, he worked at the Arms Control and Disarmament Agency in Washington. From 1969-1979, he was a professor at the Wharton School of the University of Pennsylvania where he developed the Analytic Hierarchy Process for decision-making. Inspired by world events that took place during his time at the Arms Control agency, the AHP was his answer for how to deal with weapons tradeoffs, resource allocation and



decision-making. He has written eight books devoted to the theory and applications of the Analytic Hierarchy Process (AHP) and its generalization, the Analytic Network Process (ANP) for decision making with dependence and feedback. He has also written books on Modern Nonlinear Equations, Nonlinear Mathematics, Graph Theory, The Four Color Problem, Behavioral Mathematics, Queuing Theory, Optimization in Integers, and Embracing the Future, as well as numerous papers. He has also served as a consultant to many corporations and governments. His research interests include decision-making, planning, and the analysis of neural functions. His most recent book, published in 2001, is *Creative Thinking, Problem Solving & Decision Making*. It includes a powerpoint slide presentation on a CD. The book is a rich collection of ideas, incorporating research by a growing body of researchers and practitioners, profiles of creative people, projects and products, theory, philosophy, physics and metaphysics...all explained with a liberal dose of humor. Dr. Saaty is the co-developer of AHP and ANP software and of the ANP software **SuperDecisions** for decision-making with dependence and feedback.

FOREWORD

We want this book to help people be exemplary survivors, persistent and durable, creative decision-makers and leaders, effective negotiators, doers and implementers of ideas and plans.

The Analytic Network Process (ANP) is the most comprehensive framework for the analysis of societal, governmental and corporate decisions that is available today to the decision-maker. It allows one to include all the factors and criteria, tangible and intangible that have bearing on making a best decision. The Analytic Network Process allows both interaction and feedback within clusters of elements (inner dependence) and between clusters (outer dependence). Such feedback best captures the complex effects of interplay in human society, especially when risk and uncertainty are involved.

The ANP, developed by Thomas L. Saaty, provides a way to input judgments and measurements to derive ratio scale priorities for the distribution of influence among the factors and groups of factors in the decision. Ratio scales make possible proportionate allocation of resources according to derived priorities. The well-known decision theory, the Analytic Hierarchy Process (AHP) is a special case of the ANP. Both the AHP and the ANP derive ratio scale priorities by making paired comparisons of elements on a common property or criterion. Although many decision problems are best studied through the ANP, one may wish to compare the results obtained with it to those obtained using the AHP or any other decision approach with respect to the time it took to obtain the results, the effort involved in making the judgments, and the relevance and accuracy of the results.

ANP models have three parts: the first is a strategic criteria in terms of which a decision is evaluated according to its merits of Benefits, Opportunities, Costs and Risk. Each merit provides control criteria for the second part of the decision and with each control criterion there is associated a network of influences that determine the priorities of the alternatives of the decision for that control criterion. The priorities of the merits and those of the control criteria are then used to synthesize the priorities of the alternatives to obtain the final best answer. The supermatrix and its powers are the fundamental tools needed to lay out the workings of the ANP

The ANP has been applied to a large variety of decisions: marketing, medical, political, military, social, forecasting and prediction and many others. Its accuracy of prediction is impressive in applications that have been made to economic trends, sports and other events for which the outcome later became known. Detailed case studies of applications are included in this book and also in the book, *The Analytic Network Process: Decision Making with Dependence and Feedback*, by Thomas L. Saaty.

TABLE OF CONTENTS

FOREWORD	iv
TABLE OF CONTENTS	v
COURSE OUTLINE	vii
Decision Making In Complex Environments with the ANP	vii
Course Notes	vii
Lectures, Reading and Software Assignments.....	vii
CHAPTER 1	1
AHP Basic Decision-making Concepts and Examples of Models	1
1. Introduction to Decision-making with the AHP	1
2. Creativity and Decision Making	4
3. Basic AHP Concepts and Examples	5
4. Mathematics of the Analytic Hierarchy Process	30
5. The Seven Pillars of the Analytic Hierarchy Process	32
CHAPTER 2	50
Using the SuperDecisions Software to Build AHP Hierarchical Decision Models	50
Introduction to Basic Concepts	50
Lesson One - Building a Hierarchical Model	53
Starting / Creating a New Model (File).....	53
Assessments / Pairwise Comparisons Overview.....	56
Synthesis - Getting the Results	68
Ratings Models Overview.....	69
Hierarchical Models with Sub-criteria	74
Complete Hierarchical Models	75
CHAPTER 3	78
Using the Super Decisions Software to Build ANP Models	78
Introduction to Decision Making using the Analytic Network Process (ANP)	79
Concepts of the Analytic Hierarchy Process.....	79
Concepts of the Analytic Network Process.....	80
Outline of Steps to Solve a Problem Using the ANP.....	80
Two Ways to Frame the Question when Making ANP Comparisons	82
Super Decisions Software Tutorial.....	82
Installing the ANP Software	82
Introduction to the Super Decisions Software	83
Demonstration of the Simplest Type of Feedback Network, the Bridge Model.....	84
Demo of the Hamburger Model - A Simple Network Model	90

Demonstration of a Two-layer System, the Car Purchase BCR Model.....	99
Demonstration of a Complex Three-layer System: The National Missile Defense Model (NMD)	107
Overview of Formulas for Multi-layer Models.....	119
How to Build and Get Results in a Model with Subnets- A Walkthrough	119
Creating a Model.....	120
Starting a New Model	120
Creating a sub-network	126
Completing the Sub-networks.....	126
Creating Connections or Links between Nodes	130
Making Judgments or Performing Assessments or Comparisons	130
Getting Results	135
Sensitivity.....	139
File Backup and Automatic Saving.....	143
Special ANP Software Functions and Commands.....	144
Formulas.....	144
Printing.....	149
Some Characteristics of Sub-nets.....	150
Quick Steps for doing a Complex Model with Ratings	150
CHAPTER 4	152
Applications of the ANP	152
1. Single Control Criterion Models	152
Example 1. Water Reservoir example – Optimal Level of Water in a Dam.....	152
Example 2 – Hamburger Example Re-visited.....	154
2. Complex Network Models.....	159
Example 3. The Future of the University of Pittsburgh Medical Center: Strategic Planning with the ANP.....	159
Example 4. Hong Kong and the Financial Centers in Asia in 2000	176
APPENDIX: THE ENCYCLICON – A DICTIONARY OF ANP MODELS.....	A-1

SAMPLE COURSE OUTLINE

DECISION MAKING IN COMPLEX ENVIRONMENTS WITH THE ANP

The main requirement of the course is a final project. You may work alone, or in at most groups of two, on a substantial decision problem of your choice. At the end of the course, hand in a polished executive report with an executive summary that includes a discussion of the problem and the elements of the problem, diagrams, the conclusion of the decision reached and an analysis of the results. Please hand in one printed copy of the final report along with the *SUPER DECISIONS* software model on a disk. This software allows you to attach files inside a model, so your final report in Word™, any excel and powerpoint files and other supporting material should also be attached within the model you hand in.

COURSE NOTES

BOOK (OPTIONAL)

Decision Making with Dependence and Feedback: The Analytic Network Process (ANP), 2001, by Thomas L. Saaty, RWS Publications, 4922 Ellsworth Avenue, Pittsburgh, PA 15213, Tel: 412-621-4492, Fax: 412-682-7008. It may also be available in the Pitt bookstore. There are two reference copies in the Katz library. You are not required to purchase this book. Studying it, however, is useful for a better understanding of the theory. We refer to it in the course outline as the ANP book while the course notes are referred to as the Manual. The ANP book includes additional examples.

THE SOFTWARE PACKAGE THAT WILL BE USED

The ANP software: Super Decisions –The *SUPER DECISIONS* software can be obtained free for educational and academic use by downloading it from www.creativedecisions.org and generating a serial number there.

(Shortcut: You can copy the serial number to the Windows clipboard by blocking it and pressing <Ctrl> C, then the first time you run the software you will be asked to enter a serial number. Enter it by pressing <Ctrl> V.)

LECTURES AND READING AND SOFTWARE ASSIGNMENTS

(7 week class)

PLEASE BRING YOUR LAPTOP COMPUTER TO EVERY CLASS IF POSSIBLE

LECTURE 1 AHP

Introduction to the Analytic Hierarchy Process (AHP) for decision making when the factors are independent:

- Theory of decision making using the AHP
- Examples and in-class exercise to estimate relative areas of some geometric figures
- Two AHP models will be created in class using the *SUPER DECISIONS* software; the first using relative measurement and the second using absolute measurement (or ratings with standards).

READING ASSIGNMENT

Study the tutorial for building AHP models.

Understanding how to model a decision using a hierarchy is essential in order to work later on with the more sophisticated and realistic ANP network models.

SOFTWARE ASSIGNMENT DUE TO BE TURNED IN AT SECOND CLASS PERIOD

Build an AHP model for a decision of personal interest to you: where to take a vacation, where to live, buy a house or live in an apartment, what car to buy, what job to take, etc. Do the problem in two ways as a relative model and as an absolute model. Hand in a brief write-up describing your problem. Include a description of your alternatives, screen captures from the software and a summary with analysis of the results.

LECTURE 2 ANP

1. ANP and interdependence introduction to the Analytic Network Process
2. Market share prediction models
3. Benefits, opportunities, costs and risks models

Homework Do a market share exercise structured as a network with dependence and feedback with market share (economic influence) as the underlying control criterion in making all the pairwise comparisons. Find data on the internet or elsewhere to compare your results against. Include a write-up, your model and the comparative data and its source in what you hand in.

LECTURE 3 HOW TO STRUCTURE COMPLEX DECISIONS: BENEFITS, OPPORTUNITIES, COSTS AND RISKS

1. Corporate and government decisions
 - a. National Missile Defense case
 - b. Ford Explorer tire problem case
 - c. Porsche SUV decision case

Homework Select a project from among the sample models that have been provided on the Super Decisions CD to report on during the next class.

IN-CLASS ASSIGNMENT

Select a decision problem to work on that will involve feedback. You may work alone or with one other person. This is the major project of the course. The group work begins now and continues until the end of the course when the groups present and hand in their projects.

OUT OF CLASS ASSIGNMENT FOR GROUP PROJECT

Choose a problem to work on by the next class period and be prepared to discuss it in class. Research your problem as thoroughly as possible. Each group submits a summary of what they have to do to work

out their examples to make sure they understand the decision process from strategic criteria to the synthesis of the network priorities of the alternatives under the control criteria for each of the four merits.

LECTURE 4 RESOURCE ALLOCATION

1. Resource allocation
2. Fisheries example
3. Read paper on resource allocation that will be handed out
4. Budgetary allocation

Homework

1. Do a resource allocation exercise, write up and hand in two weeks at Lecture 6.
2. Read works or suggest ways if there are no works already done to report on (about 15 minutes) in next class.
3. Elaborate on your final project topic and be prepared to talk about your project again in next class.

IN-CLASS WORK

There will be greater elaboration about using the software.

SOFTWARE ASSIGNMENT

Use the *ANP* Software to create the structure of the ANP model for your final project. The ANP model should include, if you can, all four BOCR merits.

To save time and frustration save the model frequently in a series of backup models with different names, so that you can always recover an earlier version of the model. A critical test of the soundness of your model is whether you can answer the question that would be posed in making a pairwise comparison. Making all the $n(n-1)/2$ judgments may seem very tedious but it is useful to best capture reality. There are shortcuts you can use. The minimum number of judgments required is $n-1$; for example the first row, or the diagonal, or the last column. But when using such short sets of judgments you should add a few more than the minimum required to get some redundancy. You should lock onto your project for the next two weeks and work steadily to make sure that its quality is the best possible and that the information you have would not be contestable if your final results were shown to experts in the area of the decision. Thus you must read a lot, talk a lot about the subject and make calls to knowledgeable people in the subject whenever necessary. You should include these as references at the end of your project.

LECTURE 5 AREAS WHERE PRIORITIZATION IS WIDELY USED IN BUSINESS AND SOCIETY

1. Student admissions
2. University and business school rankings
3. Fortune 500 rankings
4. Faculty promotion and tenure
5. Ranking countries for investment
6. Project prioritization and resource allocation
7. Vendor selection
8. Others

In Class Presentation

Give some details of progress on final project.

Homework

Find out best ways that people use to determine the alternatives of a decision, detailed enough for implementation and flexible enough that as time passes it is possible to move to another alternative without too much damage over time to that decision. Report on it in Lecture 6.

LECTURE 6 PLANNING AND MORPHOLOGICAL ANALYSIS

1. Using Morphological Analysis to define decision alternatives in detail
2. Forward and backward planning

LECTURE 7

Present final project to the class. This presentation and the final report on the project will determine a significant portion of your grade. Be sure to attach your final report, in Word, and any Excel files and Powerpoint files inside your software model. Be thorough in your presentation of the background material, the description of the structure of the decision and each of its components and elements and with a modicum of continuity from start to end. Do not assume that the reader really knows all that you have in mind. But you can assume that your reader knows the ANP process.

Final reports should be handed in with one printed copy and an accompanying diskette that includes the ANP model with files attached inside for any of the following that belong with the project: Excel spreadsheets, Powerpoint slides of your presentation, final report document, etc. Please make sure that the name(s) of the author(s) are on the printed copies and diskette.

Chapter 1

AHP BASIC DECISION-MAKING CONCEPTS AND EXAMPLES OF MODELS

1. INTRODUCTION TO DECISION-MAKING WITH THE AHP

Decision-making is the most central and pervasive human activity, intrinsic in our biology and done both consciously and unconsciously. We need it to survive.

Everybody makes decisions all the time, young and old, educated or uneducated, with ease or with great difficulty. Making a decision is not just a question of selecting a best alternative. Often one needs to prioritize all the alternatives for resource allocation, or to combine the strengths of preferences of individuals to form a collective preference.

Applying mathematics to decision-making calls for ways to quantify or prioritize personal or group judgments that are mostly intangible and subjective. Decision-making requires doing what is traditionally thought to be impossible, comparing apples and oranges. But we can compare apples and oranges by decomposing our preferences into the many properties that apples and oranges have, determining their importance to us, comparing and obtaining the relative preference of apples and oranges with respect to each property, and synthesizing the results to get the overall preference.

Breaking a problem down into its constituent parts or components, in the framework of a hierarchy or a feedback network, and establishing importance or priority to rank the alternatives is a comprehensive and general way to look at the problem mathematically. This kind of concern has been loosely called multicriteria decision-making.

In operations research and management science today, decision-making is essentially thought of in the focused area of research concerned with goals and criteria and how to measure and rank them. The journals have specialized editors for processing papers in the new area.

The majority of models in the literature of operations research have been concerned with single criterion decision-making. That criterion, known in optimization as an objective function, is necessarily a measurable quantity. The decision is made, for example, by maximizing dollars to maximize economic success, minimizing the amount of material used to maximize factory efficiency, or minimizing distance to minimize traveling costs.

The tangibles of today were intangibles not too far back, and how they are measured involves the use of an arbitrary unit that is replicated so many times in each reading to obtain a scale. In the end, one must make some correspondence between a real world outcome and the number from the scale.

Clearly, for many situations people will differ in what they subjectively imagine the number means despite the much talked about objectivity thought to inhere in the scale reading itself. In this sense, multicriteria decision-making looks beyond the manipulation of numbers from scales into the validity of how judgments arise and the legitimacy and accuracy of representing these judgments with numbers. This is particularly useful in making predictions of happenings and in assessing the likelihood and intensity of occurrences; and finally, it is also useful in making optimal decisions that can be easily reviewed and modified to survive the turbulence of the future environment.

In our complex world, there are usually many solutions proposed for each problem. Each of them would entail certain outcomes that are more or less desirable, more or less certain, in the short or long term, and would require different amounts and kinds of resources.

We need to set priorities on these solutions according to their effectiveness by considering their benefits, costs, risks, and opportunities, and the resources they need.

Our present complex environment calls for a new logic--a new way to cope with the myriad factors that affect the achievement of the goals and the consistency of the judgments we use to draw valid conclusions. This approach should be justifiable and appeal to our wisdom and good sense. It should not be so complex that only the educated can use it, but should serve as a unifying tool for thought in general.

There are two parts to the multicriteria problem: how to measure what is known as intangibles, and how to combine their measurements to produce an overall preference or ranking; and then, how to use it to make a decision with the best mathematics we have. Learning how to measure intangibles gave the clue to how to combine, in one framework, tangibles having different scales with each other and with intangibles.

The solution was to treat them all as intangibles and assess their priority or utility, thus producing an integrated theory of measurement across all dimensions of preference. The measurement of the intensity of preference introduces cardinal numbers, making it possible to interpret objective numbers according to one's value system and to obtain results such as constructing a group decision function from those individuals involved, contrary to what was learned from yes-no voting.

The known approaches to multicriteria decisions are few. They include: priority theory of the Analytic Hierarchy Process, Utility and Value Theory based on the use of lottery comparisons; Bayesian Theory based on probabilities; Outranking Method based on ordinal comparison of concordance and discordance; and Goal Programming--basically a modified version of Linear Programming. There are other methods that are variations of these.

Speaking of Utility and Value Theory, one is reminded of early mankind as spoken of in Greek mythology. Daedalus was a mythological Athenian inventor and artist. He was imprisoned for his heretical views and speculated about how to escape from prison. With his son, Icarus, they thought to fly out and built pairs of wings that they stuck to themselves with wax. As the son, Icarus, flapped his wings and got into the sun, it melted the wax and he fell to his death in the "Icarian Sea". When early utility theorists speculated on how to make decisions, they thought of risky situations and of gambling houses, lotteries and bets and they wrapped their theory around lottery comparisons as if lotteries and gambling have always been here. They constructed utility functions and interval scales whose numbers cannot be added or multiplied. They forgot that all human beings make decisions and people only recently learned to deal with numbers but now one needs intensive and long indoctrination to do utility theory. When there is dependence an appeal is made to Bayesian probability theory. How they combine interval scales and lottery comparisons for different people who may want to make group decisions and have to deal with inconsistency is not mathematical but is improvised. As to whether the rank of alternatives that have

already been processed should always be preserved or whether adding new alternatives, like copies and other types, should change the old ranking, utility theory has four versions of one axiom, and selected the most severe one of them. The most stringent axiom they adopted says that they should always preserve rank and as a result, they do not consider the effect of bringing in new alternatives on the ranking of the old ones in the usual mathematics of the theory. They glued themselves to the requirement that rank should always be preserved because utility theory has only one way to rank alternatives and that way turns out to be one that always preserves rank. It is a problem similar to having to do all carpentry with only a hammer. To derive a utility function, a person needs to know about numbers, and also requires a coach who knows utility theory to align his or her brain to produce a utility or value function. According to utility theory, one cannot make a good decision without adopting the assumption of rationality of that theory that demands, among other things, that people's preferences always be transitive and that rank should always be preserved and that a utility function be monotonic.

But one can make decisions more effectively today than with utility and value functions much as we fly airplanes and missiles that use modern technology instead of wax on feather wings. Paired comparisons, a biological talent we all have, are the basis of all judgments. They do not need lotteries to work. What they need are matrices, linear algebra, and further in the continuous case they need Fredholm operators to generate priorities, both for individuals and for groups based on (reciprocal) paired comparisons. By modifying the normalization operation in paired comparisons we can preserve rank when one needs to or allow it to change when one needs to.

The youngest, and mathematically most general and robust decision theory that applies paired comparisons to tangibles and intangibles equally, is the Analytic Hierarchy Process (AHP). It is a theory of measurement that has been validated through numerous applications to complex decisions around the world, and meets the challenge to solve problems in a scientifically valid way. More significantly, it is free of paradoxes, from which one reaps two salutary results. The first is that one does not have to make unrealistic assumptions to make the theory work, and the second is that the theory and its applications can be expanded to cover much wider complexity without complicating its mathematics. It may surprise the reader to know that priority functions can be used to construct utility functions without the use of lotteries.

It has also been shown that priorities can serve the role of likelihood in probability theory and that Bayes Theorem is derivable from the dependence approach of priority theory.

Finally, it has been shown that the well known Arrow Impossibility Theorem is made possible with the use of priority functions to go from individual to group preferences. Applications of the AHP have been facilitated through the use of several software programs, including Expert Choice and its extension to decision with network dependence—the *ANP* software.

Through the use of hierarchic and network structures, the AHP attempts to incorporate the objectives, criteria, actors, time frames and alternatives that have bearing on the decision. It accommodates all the factors that some people believe should be included in describing the decision problem. Their judgments are then applied to relate and compare these factors in a systematic manner that leads to priorities in the form of principal eigenvectors and eigenfunctions (and hence, ratios scales) and to the synthesis of these priorities to derive an overall priority through the use of multilinear forms.

In the end it is people's personal and collective values that need to be served. The challenge to mathematicians today is to learn about these new ideas and create and push the models and the mathematics as far as needed.

WHY WE NEED AN ORGANIZED WAY TO MAKE DECISIONS

Our three inborn talents are creative thinking, problem solving and decision making. Of these, the most basic and primitive is decision making. We need to be able to make decisions to determine which problem to solve and bring our creative thinking to provide that solution. In its most elementary form, decision making is a chemical process. There are gates in the brain operated through physics and chemistry that create the conditions for some neurons to fire and other neurons not to fire. As a result, some are excitatory and some are inhibitory. Firing and not firing is our zero-one coding and compounding from the simple to the less simple to develop complexity and meaning within context, purpose, and action. We know enough to build to the macro level from these micro workings by adopting ratio scales and showing how such decision making and synthesis leads to complex sensory impressions of seeing and hearing. We know that this is one likely path to follow. We note that plants and animals are also built biologically to be decision-makers in order to survive. They have similar gates that allow them to respond in different degrees to a variety of stimuli. It is highly doubtful that plants and animals and unschooled people have the ability and talent to laboriously construct interval scale utility functions (for every trivial decision they make) that go nowhere as far as arithmetic is concerned, but only tell one what is preferred to what as one does with ordinal numbers.

We need an organized way to make decisions because:

1. There are benefits in laying out one's own and other people's ideas in both structure and judgment. A systematic way allows us to include all the essential criteria and establish their priorities.
2. There are too many intangibles and they are difficult to remember and relate carefully.
3. We need a framework to relate the tangibles with the intangibles and assess their effects. Both objective, real world data and subjective individual or group judgments and values need to be included. How?
4. People do group decision-making and we need a way to include the participation of many individuals in making a decision. We need to know how to take advantage of the knowledge and experience of many people.
5. We need a systematic way to track a decision when fast change takes place.
6. Intuition does not always give the right guide. We need to organize our thinking in a framework with a diversity of criteria to balance off wrong intuitive hunches.
7. We need to do sensitivity analysis: if I change my mind what will the outcome be?
8. We have to capture judgment even when reasons are not easily given. We want to allow for hunches. Not all people can give sharp justification for their judgments or enter a debate about them.

Careful tradeoffs must be made among all the factors involved. This calls for a legitimate approach to quantify the judgments. Arbitrary quantification without justification of what the numbers mean can lead to meaningless number crunching.

9. A decision aid should help people to expand and sharpen their understanding. It does not necessarily mean that it must reproduce their hunches for answers. However, it should make it possible for them to see what sort of judgments they need to espouse in order for their hunches to come about.

2. CREATIVITY AND DECISION MAKING

The AHP and ANP fit into the structuring and prioritizing part of creativity known as morphological analysis. The four aspects of creativity discussed in the literature are:

1. Brainstorming – generating ideas
2. Synectics – connecting ideas
3. Lateral Thinking – expanding ideas
4. Morphological Analysis – structuring ideas

The first three of these involve generating ideas and creating relations in general, and morphological analysis is the fourth one. It is the formal and structured part of creativity. People have believed it is there but have not described it at length to show how it works. We believe hierarchical models (AHP) and networks with feedback (ANP) may be said to be morphological analysis tools.

3. BASIC AHP CONCEPTS AND EXAMPLES

People make three general types of judgments to express importance, preference, or likelihood and use them to choose the best among alternatives in the presence of environmental, social, political, and other influences. They base these judgments on knowledge in memory or from analyzing benefits, costs, and risks. From past knowledge, we sometimes can develop standards of quality, for example: from *excellent* to *above average* to *average* to *poor*, and use them to rate the alternatives one at a time. This is useful in such repetitive situations as student admissions and salary raises that must conform with established norms. Without norms one compares alternatives instead of rating them. Comparisons must fall in an admissible range of consistency. The analytic hierarchy process (AHP) includes both the rating and comparison methods. Rationality requires developing a reliable hierarchic structure or feedback network that includes criteria of various types of influence, stakeholders, and decision alternatives to determine the best choice.

PROBLEM SOLVING

Policy makers at all levels of decision making in organizations use multiple criteria to analyze their complex problems. Multicriteria thinking is used in a formal way to facilitate their decision making. Through trade-offs it clarifies the advantages and disadvantages of policy options under circumstances of risk and uncertainty. It is also a tool vital to forming corporate strategies needed for effective competition.

Nearly all of us, in one way or another, have been brought up to believe that clear-headed logical thinking is our only sure way to face and solve problems. We also believe that our feelings and our judgments must be subjected to the acid test of deductive thinking. But experience suggests that deductive thinking is not natural. Indeed, we have to practice, and for a long time, before we can do it well. Since complex problems usually have many related factors, traditional logical thinking leads to sequences of ideas that are so tangled that their interconnections are not readily discerned.

The lack of a coherent procedure to make decisions is especially troublesome when our intuition alone cannot help us to determine which of several options is the most desirable, or the least objectionable, and neither logic nor intuition are of help. Therefore, we need a way to determine which objective outweighs another, both in the near and long terms. Since we are concerned with real-life problems we must recognize the necessity for trade-offs to best serve the common interest. Therefore, this process should also allow for consensus building and compromise.

Individual knowledge and experience are inadequate in making decisions concerning the welfare and quality of life for a group. Participation and debate are needed both among individuals and between the groups affected. Here two aspects of group decision making have to be considered. The first is a rather minor complication, namely, the discussion and exchange within the group to reach some kind of consensus on the given problem. The second is of much greater difficulty. The holistic nature of the given problem necessitates that it be divided into smaller subject-matter areas within which different groups of experts determine how each area affects the total problem. A large and complex problem can rarely be decomposed simply into a number of smaller problems whose solutions can be combined into an overall answer. If this process is successful, one can then reconstruct the initial question and review the

proposed solutions. A last and often crucial disadvantage of many traditional decision-making methods is that they require specialized expertise to design the appropriate structure and then to embed the decision-making process in it.

A decision-making approach should have the following characteristics:

- be simple in construct,
- be adaptable to both groups and individuals,
- be natural to our intuition and general thinking,
- encourage compromise and consensus building, and
- not require inordinate specialization to master and communicate.

In addition, the details of the processes leading up to the decision-making process should be easy to review.

At the core of the problems that our method addresses is the need to assess the benefits, the costs, and the risks of the proposed solutions. We must answer such questions as the following: Which consequences weigh more heavily than others? Which aims are more important than others? What is likely to take place? What should we plan for and how do we bring it about? These and other questions demand a multicriteria logic. It has been demonstrated over and over by practitioners who use the theory discussed in this paper, that multicriteria logic gives different and often better answers to these questions than ordinary logic and does it efficiently.

To make a decision one needs various kinds of knowledge, information, and technical data. These concern

- details about the problem for which a decision is needed,
- the people or actors involved,
- their objectives and policies,
- the influences affecting the outcomes, and
- the time horizons, scenarios, and constraints.

The set of potential outcomes or alternatives from which to choose are the essence of decision making. In laying out the framework for making a decision, one needs to sort the elements into groupings or clusters that have similar influences or effects. One must also arrange them in some rational order to trace the outcome of these influences. Briefly, we see decision making as a process that involves the following steps:

- (1) Structure a problem with a model that shows the problem's key elements and their relationships.
- (2) Elicit judgments that reflect knowledge, feelings, or emotions.
- (3) Represent those judgments with meaningful numbers.
- (4) Use these numbers to calculate the priorities of the elements of the hierarchy.
- (5) Synthesize these results to determine an overall outcome.
- (6) Analyze sensitivity to changes in judgment.

The decision-making process described in this paper meets these criteria. I call it the analytic hierarchy process (AHP). The AHP is about breaking a problem down and then aggregating the solutions of all the subproblems into a conclusion. It facilitates decision making by organizing perceptions, feelings, judgments, and memories into a framework that exhibits the forces that influence a decision. In the simple and most common case, the forces are arranged from the more general and less controllable to the more specific and controllable. The AHP is based on the innate human ability to make sound judgments

about small problems. It has been applied in a variety of decisions and planning projects in nearly 20 countries.

Here *rationality* is

- Focusing on the goal of solving the problem;
- Knowing enough about a problem to develop a complete structure of relations and influences;
- Having enough knowledge and experience and access to the knowledge and experience of others to assess the priority of influence and dominance (importance, preference, or likelihood to the goal as appropriate) among the relations in the structure;
- Allowing for differences in opinion with an ability to develop a best compromise.

How to Structure a Hierarchy

Perhaps the most creative part of decision making and one that has a significant effect on the outcome is modeling the problem. In the AHP, a problem is structured as a hierarchy. This is then followed by a process of prioritization. Prioritization involves eliciting judgments in response to questions about the dominance of one element over another when compared with respect to a property. The basic principle to follow in creating this structure is always to see if one can answer the following question: Can I compare the elements on a lower level using some or all of the elements on the next higher level as criteria or attributes of the lower level elements?

We have already discussed the subject of forming hierarchies and need to only summarize it in two sentences to refresh the reader's memory for what is to come below. A useful way to proceed in structuring a decision is to come down from the goal as far as one can by decomposing it into the most general and most easily controlled factors, on down to where they become more specific. One can then go up from the alternatives beginning with the simplest subcriteria that they must satisfy and aggregating the subcriteria into generic higher level criteria until the levels of the two processes are linked in such a way as to make pairwise comparisons at every level possible.

JUDGMENTS AND COMPARISONS - THE FUNDAMENTAL SCALE

A judgment or comparison is the numerical representation of a relationship between two elements that share a common parent. The set of all such judgments can be represented in a square matrix in which the set of elements is compared with itself. Each judgment represents the dominance of an element in the column on the left over an element in the row on top. It reflects the answers to two questions: which of the two elements is more important with respect to a higher level criterion, and how strongly, using the 1—9 scale shown in Table 1 for the element on the left over the element at the top of the matrix. If the element on the left is less important than that on the top of the matrix, we enter the reciprocal value in the corresponding position in the matrix.

Table 1. The Fundamental Scale.

Intensity of Importance	Definition	Explanation
1	Equal Importance	Two activities contribute equally to the objective
3	Moderate importance	Experience and judgment slightly favor one activity over another
5	Strong importance	Experience and judgment strongly favor one activity over another
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation
2,4,6,8	For compromise between the above values	Sometimes one needs to interpolate a compromise judgment numerically because there is no good word to describe it.
Reciprocals of above	If activity i has one of the above nonzero numbers assigned to it when compared with activity j , then j has the reciprocal value when compared with i	A comparison mandated by choosing the smaller element as the unit to estimate the larger one as a multiple of that unit.
Rationals	Ratios arising from the scale	If consistency were to be forced by obtaining n numerical values to span the matrix
1.1-1.9	For tied activities	When elements are close and nearly indistinguishable; moderate is 1.3 and extreme is 1.9.

It is important to note that the lesser element is always used as the unit and the greater one is estimated as a multiple of that unit. From all the paired comparisons we calculate the priorities and exhibit them on the right of the matrix. For a set of n elements in a matrix one needs $n(n-1)/2$ comparisons because there are n 1's on the diagonal for comparing elements with themselves and of the remaining judgments, half are reciprocals. Thus we have $(n^2-n)/2$ judgments. In some problems one may elicit only the minimum of $n-1$ judgments.

In Table 1 the scale of absolute numbers is shown that is used to assign numerical values to judgments. Judgments are made by comparing two elements, holding the smaller as the unit and assigning the larger a value from this scale that is closest to what you estimate is its multiple of the smaller.

GROUPING TO COMPARE THE VERY SMALL WITH THE VERY LARGE

There is a very good reason why one should use the absolute scale 1-9. If you make paired comparisons of two alternatives, you determine which is the smaller and estimate how many times (multiples) of it the larger one is. If they are too far apart like comparing a grape with a water melon, one would make such a bad mistake that it is not worth while to make the comparison; however, if one compares a grape with a plum and a small apple in one set and then again use the small apple with a large apple and a grapefruit, and for a third time separately compare the grapefruit with a melon and a water melon one has a comparison of the grape with the watermelon. In each set of comparisons one uses the 1-9 scale. In the second comparison set one divides by the weight of the small apple so it gets a "one" and multiplies by its weight in the first comparison so it has the same weight relative to the grape and the large apple and the

grapefruit are also multiplied by this weight of the small apply from the first comparison. Again if you divide the third set of comparisons by the weight of the grapefruit and multiply them by its weight from the second comparisons (adjusted as explained above) one gets the relation of the watermelon to the grape, by simply dividing by it to see how many grapes make a watermelon. If you understand this you would understand half the story why limiting the scale gets you better precision in the final answer. You do not want to make it 1-3 or 1-5 etc. because you would do much more work. In addition, people are observed to have this ability to compare homogeneous objects that are within the same order of magnitude. I believe that this order of magnitude was not chosen arbitrarily but came from observing what we can do. I have applied 1-9 to compare many physical phenomena (lengths, weights, distances, brightness, etc.) and got excellent results. One cannot use an arbitrary scale because one is comparing homogeneous elements. Assigning any numbers would be like giving numerical judgments without thinking.

THE HOSPICE PROBLEM

Westmoreland County Hospital in Western Pennsylvania, like hospitals in many other counties around the nation, has been concerned with the costs of the facilities and manpower involved in taking care of terminally ill patients. Normally these patients do not need as much medical attention as do other patients. Those who best utilize the limited resources in a hospital are patients who require the medical attention of its specialists and advanced technology equipment — whose utilization depends on the demand of patients admitted into the hospital. The terminally ill need medical attention only episodically. Most of the time such patients need psychological support. Such support is best given by the patient's family, whose members are able to supply the love and care the patients most need. For the mental health of the patient, home therapy is a benefit. From the medical standpoint, especially during a crisis, the hospital provides a greater benefit. Most patients need the help of medical professionals only during a crisis. Some will also need equipment and surgery. The planning association of the hospital wanted to develop alternatives for caring for terminally ill patients and to choose the best one considering various criteria from the standpoint of the patient, the hospital, the community, and society at large. In this problem, we need to consider the costs and benefits of the decision. Costs includes economic costs and all sorts of intangibles, such as inconvenience and pain. Such disbenefits are not directly related to benefits as their mathematical inverses, because patients infinitely prefer the benefits of good health to these intangible disbenefits. To study the problem, one needs to deal with benefits and with costs separately.

APPROACHING THE PROBLEM

We met with representatives of the planning association for several hours to decide on the best alternative. To make a decision by considering benefits and costs, one must first answer the question: In this problem, do the benefits justify the costs? If they do, then either the benefits are so much more important than the costs that the decision is based simply on benefits, or the two are so close in value that both the benefits and the costs should be considered. Then we use two hierarchies for the purpose and make the choice by forming ratios of the priorities of the alternatives (benefits b_i /costs c_i) from them. One asks which is most beneficial in the benefits hierarchy (Figure 1) and which is most costly in the costs hierarchy (Figure 2). If the benefits do not justify the costs, the costs alone determine the best alternative —that which is the least costly. In this example, we decided that both benefits and costs had to be considered in separate hierarchies. In a risk problem, a third hierarchy is used to determine the most desired alternative with respect to all three: benefits, costs, and risks. In this problem, we assumed risk to be the same for all contingencies. Whereas for most decisions one uses only a single hierarchy, we constructed two hierarchies for the hospice problem, one for benefits or gains (which model of hospice care yields the greater benefit) and one for costs or pains (which model costs more).

The planning association thought the concepts of benefits and costs were too general to enable it to make a decision. Thus, we and the planners further subdivided each (benefits and costs) into detailed subcriteria to enable the group to develop alternatives and to evaluate the finer distinctions the members perceived between the three alternatives. The alternatives were to care for terminally ill patients at the hospital, at home, or partly at the hospital and partly at home.

As usual with the AHP, in both the cost and the benefits models, we compared the criteria and subcriteria according to their relative importance with respect to the parent element in the adjacent upper level. For example, in the first matrix of comparisons of the three benefits criteria with respect to the goal of choosing the best hospice alternative, recipient benefits are moderately more important than institutional benefits and are assigned the absolute number 3 in the (1,2) or first-row second-column position. Three signifies three times more. The reciprocal value is automatically entered in the (2,1) position, where institutional benefits on the left are compared with recipient benefits at the top. Similarly a 5, corresponding to strong dominance or importance, is assigned to recipient benefits over social benefits in the (1,3) position, and a 3, corresponding to moderate dominance, is assigned to institutional benefits over social benefits in the (2,3) position with corresponding reciprocals in the transpose positions of the matrix.

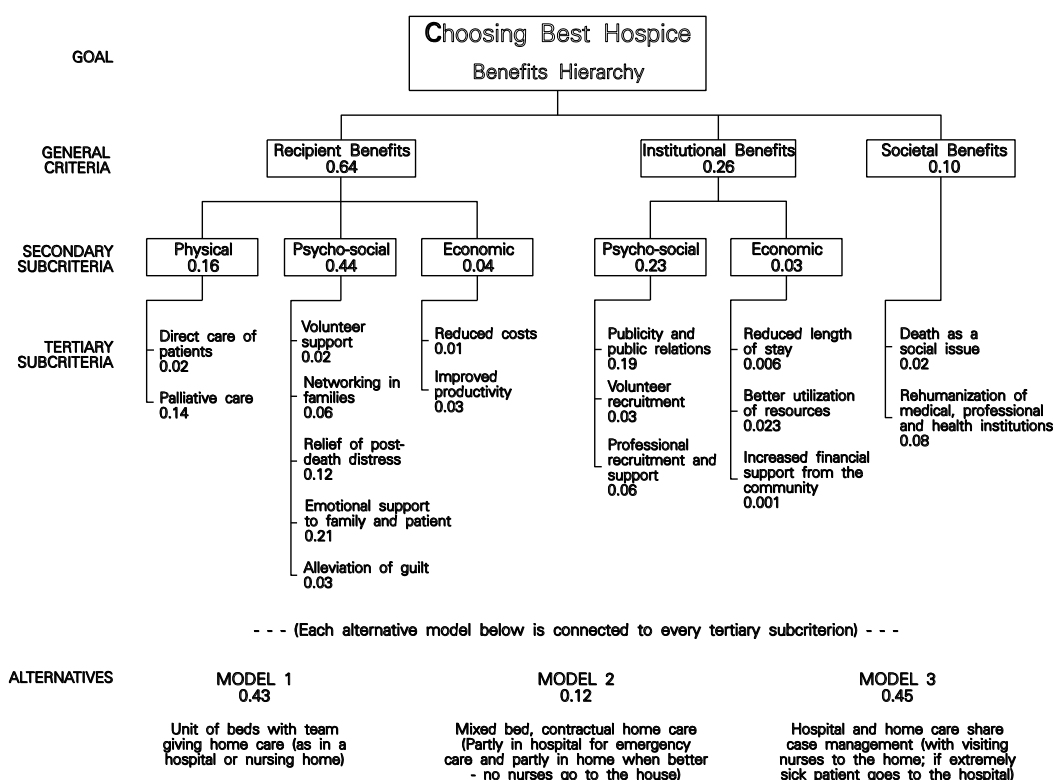


Figure 1. Hospice benefits hierarchy

For each of the two hierarchies, benefits and costs, the goal clearly had to be choosing the best hospice. We placed this goal at the top of each hierarchy. Then the group discussed and identified overall criteria for each hierarchy; these criteria need not be the same for the benefits as for the costs.

The two hierarchies are fairly clear and straightforward in their description. They descend from the more general criteria in the second level to secondary subcriteria in the third level and then to tertiary subcriteria in the fourth level on to the alternatives at the bottom or fifth level.

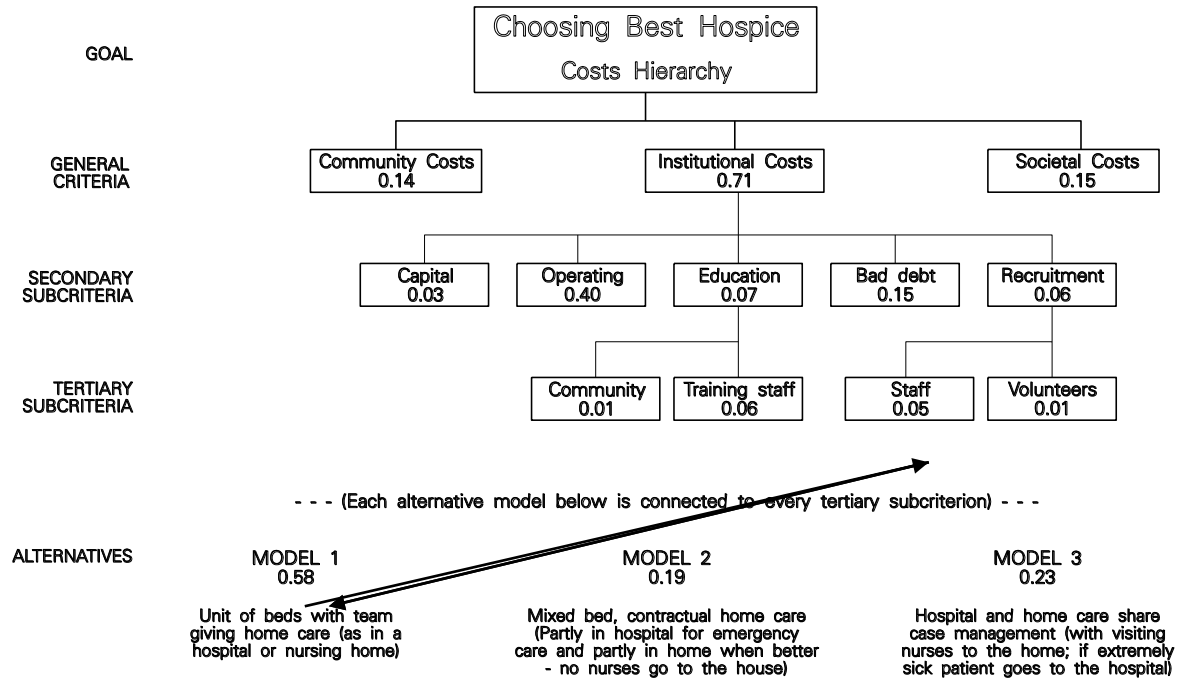


Figure 2. Hospice costs hierarchy

At the general criteria level, each of the hierarchies, benefits or costs, involved three major interests. The decision should benefit the recipient, the institution, and society as a whole, and their relative importance is the prime determinant as to which outcome is more likely to be preferred. We located these three elements on the second level of the benefits hierarchy. As the decision would benefit each party differently and the importance of the benefits to each recipient affects the outcome, the group thought that it was important to specify the types of benefit for the recipient and the institution. Recipients want physical, psycho-social and economic benefits, while the institution wants only psycho-social and economic benefits. We located these benefits in the third level of the hierarchy. Each of these in turn needed further decomposition into specific items in terms of which the decision alternatives could be evaluated. For example, while the recipient measures economic benefits in terms of reduced costs and improved productivity, the institution needed the more specific measurements of reduced length of stay, better utilization of resources, and increased financial support from the community. There was no reason to decompose the societal benefits into third level subcriteria, hence societal benefits connects directly to the fourth level. The group considered three models for the decision alternatives, and located them on the bottom or fifth level of the hierarchy: In Model 1, the hospital provided full care to the patients; In Model 2, the family cares for the patient at home, and the hospital provides only emergency treatment (no nurses go to the house); and in Model 3, the hospital and the home share patient care (with visiting nurses going to the home).

In the costs hierarchy there were also three major interests in the second level that would incur costs or pains: community, institution, and society. In this decision the costs incurred by the patient were not included as a separate factor. Patient and family could be thought of as part of the community. We thought decomposition was necessary only for institutional costs. We included five such costs in the third level: capital costs, operating costs, education costs, bad debt costs, and recruitment costs. Educational costs apply to educating the community and training the staff. Recruitment costs apply to staff and volunteers. Since both the costs hierarchy and the benefits hierarchy concern the same decision, they both have the same alternatives in their bottom levels, even though the costs hierarchy has fewer levels.

Table 2. Comparing Benefits of the Hospice Major Benefit Criteria

Choosing Best Hospice	Recipient Benefits	Institutional Benefits	Social Benefits	Priorities
Recipient Benefits	1	3	5	.64
Institutional Benefits	1/3	1	3	.26
Societal Benefits	1/5	1/3	1	.11

C.R. = .033

When the judgments are inconsistent, the decision-maker may not know where the greatest inconsistency is. The AHP can show one by one in sequential order which judgments are the most inconsistent, and that suggests the value that best improves consistency. However, this recommendation may not necessarily lead to a more accurate set of priorities that correspond to some underlying preference of the decision-makers. Greater consistency does not imply greater accuracy and one should go about improving consistency (if one can, given the available knowledge) by making slight changes compatible with one's understanding. If one cannot reach an acceptable level of consistency, one should gather more information or reexamine the framework of the hierarchy.

Under each matrix, we have indicated a consistency ratio (CR) comparing the inconsistency of the set of judgments in that matrix with what it would be if the judgments and the corresponding reciprocals were taken at random from the scale.

Priorities are numerical ranks measured on a ratio scale. If some element has a value of .6 and another a value of .3, because these are ratio scale numbers, the first is twice as large as the second. An example of a ratio scale is the scale used to measure weight. The ratio of these weights is the same in pounds and in kilograms. Here one scale is just a constant multiple of the other. The object of evaluation is to elicit judgments concerning relative importance of the elements of the hierarchy to create scales of priority of influence.

Because the benefit priorities of the alternatives at the bottom level belong to a ratio scale and their cost priorities also belong to a ratio scale, and since the product or quotient (but not the sum or the difference) of two ratio scales is also a ratio scale, to derive the answer we divide the benefit priority of each alternative by its cost priority. We then choose the alternative with the largest of these ratios. It is also possible to allocate a resource proportionately among the alternatives.

Local priorities are derived from judgments with respect to a single criterion.

Global priorities are derived from multiplication by the priority of the criterion. Overall priorities for the alternatives are derived by adding their global priorities. The local priorities are listed on the right of each matrix.

The process is repeated in all the matrices by asking the appropriate dominance or importance question. For example, for the matrix comparing the subcriteria of the parent criterion institutional benefits (Table 3), psycho-social benefits are regarded as very strongly more important than economic benefits, and 7 is entered in the (1,2) position and 1/7 in the (2,1) position.

In comparing the three models for patient care, we asked members of the planning association which model they preferred with respect to each of the covering or parent secondary criterion in level 3 or with respect to the tertiary criteria in level 4. For example, for the subcriterion direct care (located on the left-most branch in the benefits hierarchy), we obtained a matrix of paired comparisons (Table 4) in which Model 1 is

preferred over Models 2 and 3 by 5 and 3 respectively and Model 3 is preferred by 3 over Model 2. The group first made all the comparisons using semantic terms from the fundamental scale and then translated them to the corresponding numbers.

Table 3. Pairwise Comparing the Hospice Institutional Benefits

Institutional benefits	Psycho-social	Economic	Priorities
Psycho-social	1	7	.875
Economic	1/7	1	.125

C.R. = .000

Table 4. Comparing Hospice Alternatives under patient care

Direct care of patient	Model I	Model II	Model III	Priorities
Model I - Unit/Team	1	5	3	.64
Model II - Mixed/Home Care	1/5	1	1/3	.10
Model III - Case Management	1/3	3	1	.26

C.R. = .033

For the costs hierarchy, we again illustrate with three matrices. First the group compared the three major cost criteria and provided judgments in response to the question: which criterion is a more important determinant of the cost of a hospice model? Table 5 shows the judgments obtained.

Table 5. Comparing Major Cost Criteria in Hospice example

Choosing best hospice (costs)	Community	Institutional	Societal	Priorities
Community Costs	1	1/5	1	.14
Institutional Costs	5	1	5	.71
Societal Costs	1	1/5	1	.14

C.R. = .000

The group then compared the subcriteria under institutional costs and obtained the importance matrix shown in Table 6.

Table 6. Comparing Hospice Institutional Costs Subcriteria (Which is more costly?)

Institutional costs	Capital	Operating	Education	Bad Debt	Recruitment	Priorities
Capital	1	1/7	1/4	1/7	1	.05
Operating	7	1	9	4	5	.57
Education	4	1/9	1	1/2	1	.10
Bad Debt	7	1/4	2	1	3	.21
Recruitment	1	1/5	1	1/3	1	.07

C.R. = .08

Finally we compared the three models to find out which incurs the highest cost for each criterion or subcriterion. Table 7 shows the results of comparing them with respect to the costs of recruiting staff.

Table 7. Comparing Hospice Alternatives for Institutional Costs

Institutional costs for recruiting staff	Model I	Model II	Model III	Priorities
Model I - Unit/Team	1	4	4	.66
Model II - Mixed/Home Care	1/4	1	1	.17
Model III - Case Management	1/4	1	1	.17

C.R. = .000

As shown in Table 8, we divided the benefits priorities by the costs priorities for each alternative to obtain the best alternative, model 3, the one with the largest value for the ratio.

Model 3 has the largest ratio scale values of benefits to costs in both the distributive and ideal modes, and the hospital selected it for treating terminal patients. This need not always be the case. In this case, there is dependence of the personnel resources allocated to the three models because some of these resources would be shifted based on the decision. Therefore the distributive mode is the appropriate method of synthesis. If the alternatives were sufficiently distinct with no dependence in their definition, the ideal mode would be the way to synthesize. For distinction between the distributive and ideal modes of hierarchic synthesis see the next section.

Marginal analysis was performed to determine where the hospital should allocate additional resources for the greatest marginal return. To perform marginal analysis, we first ordered the alternatives by increasing cost priorities and then formed the benefit-to-cost ratios corresponding to the smallest cost, followed by the ratios of the differences of successive benefits to costs. If this difference in benefits is negative, the new alternative is dropped from consideration and the process continued. The alternative with the largest marginal ratio is then chosen. For the costs and corresponding benefits from the synthesis rows in Table 8 we obtained:

	Model I	Model II	Model III
Costs	.20	.21	.59
Benefits	.12	.45	.43
Marginal Ratios	$.12/.20 = 0.60$	$(.45-.12)/(.21-.20) = 33$	$(.43-.45)/(.59-.21) = -0.05$

Table 8. The Benefit/Cost Ratios are obtained for Both the Distributive and Ideal Modes.

Benefits	Priorities	Distributive Mode			Ideal Mode		
		Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
Direct Care of Patient	.02	0.64	0.10	0.26	1.000	0.156	0.406
Palliative Care	.14	0.64	0.10	0.26	1.000	0.156	0.406
Volunteer Support	.02	0.09	0.17	0.74	0.122	0.230	1.000
Networking in Families	.06	0.46	0.22	0.32	1.000	0.478	0.696
Relief of Post Death Stress	.12	0.30	0.08	0.62	0.484	0.129	1.000
Emotional Support of Family and Patient	.21	0.30	0.08	0.62	0.484	0.129	1.000

Alleviation of Guilt	.03	0.30	0.08	0.62	0.484	0.129	1.000
Reduced Economic Costs for Patient	.01	0.12	0.65	0.23	0.185	1.000	0.354
Improved Productivity	.03	0.12	0.27	0.61	0.197	0.443	1.000
Publicity and Public Relations	.19	0.63	0.08	0.29	1.000	0.127	0.460
Volunteer Recruitment	.03	0.64	0.10	0.26	1.000	0.156	0.406
Professional Recruitment and Support	.06	0.65	0.23	0.12	1.000	0.354	0.185
Reduced Length of Stay	.006	0.26	0.10	0.64	0.406	0.406	1.000
Better Utilization of Resources	.023	0.09	0.22	0.69	0.130	0.130	1.000
Increased Monetary Support	.001	0.73	0.08	0.19	1.000	1.000	0.260
Death as a Social Issue	.02	0.20	0.20	0.60	0.333	0.333	1.000
Rehumanization of Institutions	.08	0.24	0.14	0.62	0.387	0.226	1.000
Synthesis		0.428	0.121	0.451	0.424	0.123	0.453
Costs							
Community Costs	.14	0.33	0.33	0.33	1.000	1.000	1.000
Institutional Capital Costs	.03	0.76	0.09	0.15	1.000	0.118	0.197
Institutional Operating Costs	.40	0.73	0.08	0.19	1.000	0.110	0.260
Institutional Costs for Educating the Community	.01	0.65	0.24	0.11	1.000	0.369	0.169
Institutional Costs for Training Staff	.06	0.56	0.32	0.12	1.000	0.571	0.214
Institutional Bad Debt	.15	0.60	0.20	0.20	1.000	0.333	0.333
Institutional Costs of Recruiting Staff	.05	0.66	0.17	0.17	1.000	0.258	0.258
Institutional Costs of Recruiting Volunteers	.01	0.60	0.20	0.20	1.000	0.333	0.333
Societal Costs	.15	0.33	0.33	0.33	1.000	1.000	1.000
Synthesis		0.583	0.192	0.224	0.523	0.229	0.249
Benefit/Cost Ratio		0.734	0.630	2.013	0.811	0.537	1.819

The third alternative is best. It is not a contender for resources because its marginal return is negative. The second alternative is best for investment because of its marginal return, even though it has the lowest benefit to cost ratio. In fact, in addition to adopting the third model, the hospital management chose the second model of hospice care for development to support the third model in case of shortage of personnel.

THE DISTRIBUTIVE AND IDEAL MODES

Table 8 in the Hospice Example above shows two ways or modes of synthesizing the local priorities of the alternatives using the global priorities of their parent criteria: The distributive mode and the ideal mode. In the distributive mode, the weights of the alternatives sum to one. It is used when there is dependence among the alternatives and a unit priority is distributed among them. The ideal mode is used to obtain the single best alternative regardless of what other alternatives there are. In the ideal mode, the local priorities of the alternatives are divided by the largest value among them. This is done for each criterion; for each criterion one alternative becomes an ideal with value one. In both modes, the local priorities are weighted by the global priorities of the parent criteria and synthesized and the benefit-to-cost ratios formed. In this case, both modes lead to the same outcome for hospice, which is model 3. As we shall see below, we need both modes to deal with the effect of adding (or deleting) alternatives on an already ranked set.

In the distributive mode the weight of a criterion reflects the importance that the decision maker attaches to the dominance of each alternative relative to all other alternatives under that criterion. In the ideal mode the importance of the weight of a criterion reflects the importance that the decision maker attaches to the relative performance of an alternative to some benchmark alternative. To choose the relevant mode one asks: Do you want to choose an alternative that is better relative to the others (distributive) or do you want the best of the alternatives (ideal). The better is only best in the given collection. It may in fact not be such a good alternative.

RESOLUTION OF THE RANK PRESERVATION ISSUE

Early developers of utility theory axiomatically ruled that introducing alternatives, particularly "irrelevant" ones, should not cause rank reversal [Luce and Raiffa 1957]. A theory that rates alternatives one at a time, as in the absolute measurement salary-raise example given before, assumes the existence of past standards established by experts for every decision problem and would thus assume that every decision can be made by rating each alternative by itself without regard to any other alternative and would

inexorably preserve rank. But if past standards are inapplicable to new problems and if experts are not sufficiently familiar with the domain of a decision to establish standards and the environment changes rapidly, an insistence on making decisions based on standards will only force the organization to shift its efforts from solving the problem to updating its standards. For example, practitioners have improvised many techniques to relate standards defined by utility functions in the context of a specific decision problem. Connecting theory to practice is important but often difficult. We need to distinguish between fixing the axioms of a decision theory to be followed strictly in all situations and learning and revising in the process of making a decision. The rank preservation axioms of utility theory and the AHP parallel the axioms of the classical frequentist method of statistics and Bayesian theory. Bayesian theory violates the axioms of statistics in updating prediction by including information from a previous outcome, a process known as learning. When we integrate learning with decision making, we question some of the static basic axioms of utility theory.

The AHP avoids this kind of formulation and deals directly with paired comparisons of the priority of importance, preference, or likelihood (probability) of pairs of elements in terms of a common attribute or criterion represented in the decision hierarchy. We believe that this is the natural (but refined) method that people followed in making decisions long before the development of utility functions and before the AHP was formally developed.

The major objection once raised against the AHP by practitioners of utility theory and addressed in the AHP literature and software Expert Choice has been this issue of rank reversal. The issues of rank reversal and preference reversal have been much debated in the literature regarding its occurrence in utility theory [Grether and Plott 1979; Hershey and Schoemaker 1980; Pommerehne, Schneider, and Zweifel 1982; Saaty 1994, Chapter 5; Tversky and Simonson 1993, Tversky, Slovic, and Kahneman 1990].

Regularity is a condition of choice theory that has to do with rank preservation. R. Corbin and A. Marley [1974] provide a utility theory example of rank reversal. It "concerns a lady in a small town, who wishes to buy a hat. She enters the only hat store in town, and finds two hats, A and B, that she likes equally well, and so might be considered equally likely to buy. However, now suppose that the sales clerk discovers a third hat, C, identical to B. Then the lady may well choose hat A for sure (rather than risk the possibility of seeing someone wearing a hat just like hers), a result that contradicts regularity." Utility theory has no clear analytical answer to this paradox nor to famous examples having to do with phantom alternatives and with decoy alternatives that arise in the field of marketing [Saaty 1994]. When many irrelevant alternatives are added, their very number can become a serious factor that influences the outcome, a fact often ignored by utility theory. Again, when rating poor alternatives to choose the best one and then bringing a good alternative, it can influence what is ranked best -- a less controversial observation.

Because of such examples, it is clear that one cannot simply use one procedure for every decision problem because that procedure would either preserve or not preserve rank. Nor can one introduce new criteria that indicate the dependence of the alternatives on information from each new alternative that is added. In the AHP, this issue has been resolved by adding the ideal mode to the normalization mode in relative measurement. The ideal mode prevents an alternative that is rated low or "irrelevant" on all the criteria from affecting the rank of higher rated alternatives.

In the AHP, we have one way to allow rank to change, and two ways to preserve rank as follows:

- (1) We can allow rank to reverse by using the distributive mode of the relative measurement approach of the AHP.

- (2) We can preserve rank in the case of irrelevant alternatives by using the ideal mode of the AHP relative measurement approach.
- (3) We can preserve rank absolutely by using the absolute measurement mode of the AHP.

As a recap, in relative measurement, we use normalization by dividing by the sum of the priorities of the alternatives to define the distributive mode. In this mode, we distribute the unit value assigned to the goal of a decision proportionately among the alternatives through normalization. When we add a new alternative, it takes its share of the unit from the previously existing alternatives. This mode allows for rank reversal because dependence exists among the alternatives, which is attributable to the number of alternatives and to their measurements values and which is accounted for through normalization. For example, multiple copies of an alternative can affect preference for that alternative in some decisions. We need to account for such dependence in allocating resources, in voting and in distributing resources among the alternatives.

In the ideal mode, we would simply compare a new alternative with the ideal (with the weight of one), and it would fall below or above the ideal and could itself become the ideal. As a result, an alternative that falls far below the ideal on every criterion cannot affect the rank of the best chosen alternative. Using absolute measurement, we rate alternatives one at a time with respect to each criterion, and this process cannot give rise to rank reversal.

An experiment was conducted involving 64,000 hierarchies with priorities assigned randomly to criteria and to alternatives to test the number of times the best choice obtained by the distributive and ideal modes coincided with each other. It turns out that the two methods yield the same top alternative 92 percent of the time. Similar results were obtained for the top two alternatives [Saaty and Vargas 1993a].

A SUSTAINABLE METROPOLITAN GROWTH PROBLEM

In the following illustration there are nine alternatives evaluated in terms of benefit, cost and risk impacts to sustain metropolitan growth. Here again one asks which is more risky (or hazardous) and in the end one divides the benefit to cost ratios by the overall risk priorities of each alternative. The alternatives are: build infrastructure (of roads, telecommunication, banking, etc.), improve quality of life, reduce taxes, status quo (or do nothing), provide land for expansion, foster increased tourism, expand port, support industries and improve regional connections. The criteria in each of the three hierarchies of Figure 3 are different and will not be described in the text because they are laid out in the diagrams. The distributive mode was used to synthesize the priorities of the alternatives. We only give the overall inconsistency in the case of the benefits hierarchy. To save space we have not given all the matrices of paired comparisons. We only illustrate with the matrices of judgments for the criteria of each hierarchy, Tables 9,10 and 12, and only one set of subcriteria, under social costs, in Table 11.

THE CRITERIA AND SUBCRITERIA MATRICES

Table 9. Comparing Metropolitan Growth Benefits Criteria

Benefits	Create jobs	Increase tax base	Quality of life	Improve infrastructure	Priorities
Create jobs	1	4	1.3	3	0.431
Increase tax base	1/4	1	1/3	1/2	0.097
Quality of life	1/1.3	3	1	1.5	0.294
Improve infrastructure	1/3	2	1/1.5	1	0.177

C.R. = 0.013

Which criterion is a more important benefit? In comparing the alternatives with respect to each criterion in a separate matrix one asks: Which alternative yields greater benefit with respect to that criterion?

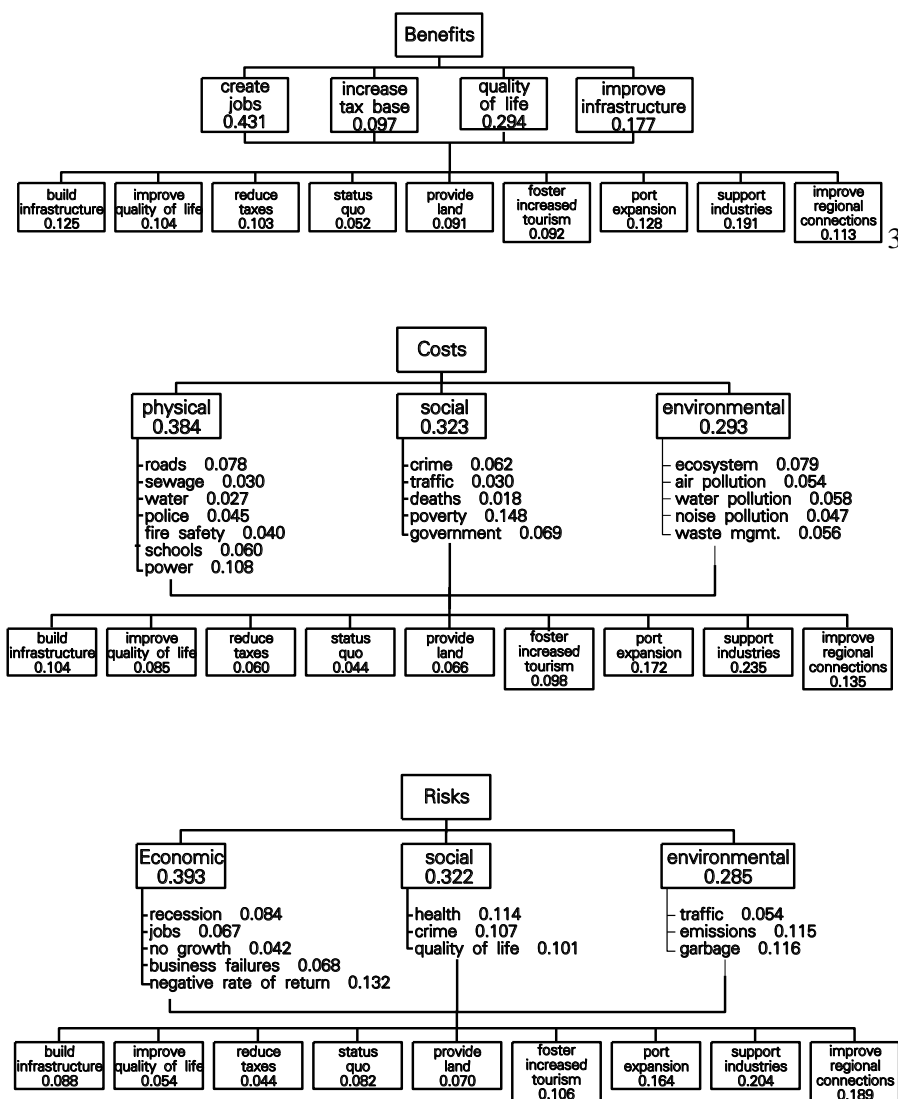


Figure 3. Metropolitan Growth Benefits, Costs and Risks Hierarchies

Table 10. Comparing Metropolitan Growth Costs Criteria

Costs	Physical	Social	Environmental	Priorities
Physical	1	1.3	1.2	0.384
Social	1/1.3	1	1.2	0.323
Environmental	1/1.2	1/1.2	1	0.293

C.R. = 0.007

Which criterion is a more important cost? The Physical criterion. Which alternative incurs greater cost with respect to that criterion? Evaluate the alternatives with respect to each criterion.

Table 11. Comparing Metropolitan Growth Social Cost Subcriteria

Societal Costs	Crime	Traffic	Deaths	Poverty	Government	Priorities
Crime	1	3	4	1/3	1/1.4	0.192
Traffic	1/3	1	1.67	1/4	1/2	0.091
Deaths	1/4	1/1.67	1	1/5	1/5	0.056
Poverty	3	4	5	1	3	0.448
Government	1.4	2	5	1/3	1	0.213

C.R. = 0.041

Which subcriterion is the most important social cost? The Poverty subcriterion.

Table 12. Comparing Metropolitan Growth Risks Criteria

Risk	Economic	Social	Environmental	Priorities
Economic	1	1.3	1.3	0.393
Social	1/1.3	1	1.2	0.322
Environmental	1/1.3	1.2	1	0.285

C.R. = 0.004

Which criterion is the riskiest? The Economic criterion. Which alternative incurs greater risk with respect to that criterion? Evaluate the alternatives with respect to each of them.

Table 13. Combining Benefits, Costs and Risks

	Industry	Port	Infra	Region	Life	Tax	Tourism	Land	Status
Benefit	0.191	0.128	0.125	0.113	0.104	0.103	0.092	0.091	0.052
Cost	0.235	0.172	0.104	0.135	0.085	0.060	0.098	0.066	0.044
B/C	0.813	0.744	1.202	0.985	1.224	1.717	0.939	1.379	1.182
Mgnl B/C	1.000	0.044	-	0.180	0.040	-	0.031	1.773	1.182
Risk	0.204	0.164	0.088	0.189	0.054	0.044	0.106	0.070	0.082
B/CR	3.984	4.537	13.659	5.212	22.667	39.023	8.858	19.70	14.415

In Table 13, we see that the ranking of the alternatives according to benefit/cost x risk ratios descends from reduce taxes, improve quality of life, provide land, maintain status quo, build infrastructure and so on. One could allocate resources proportionately to these ratio scale outcomes. Three possible ways to introduce risk in the AHP are: 1) through a separate hierarchy as above; 2) through risk scenarios in a forecasting hierarchy; or 3) through risk criteria if one has already decided to go ahead with that decision problem, just as one includes low cost in a car buying hierarchy.

ABSOLUTE MEASUREMENT

Cognitive psychologists have recognized for some time that people are able to make two kinds of comparisons - absolute and relative. In absolute comparisons, people compare alternatives with a standard in their memory that they have developed through experience. In relative comparisons, they compared alternatives in pairs according to a common attribute, as we did throughout the hospice example.

People use absolute measurement (sometimes also called rating) to rank independent alternatives one at a time in terms of rating intensities for each of the criteria. In absolute measurement a hierarchy is developed in the usual way down to the level of criteria or subcriteria. The criteria or subcriteria are further subdivided into a level for intensities. Each criterion has ratings listed under it. An example would be to take a criterion of cost and list under it very high, high, average, and low. These are ratings that are then prioritized to determine their relative importance. The type and number of ratings for each criterion may be different. An intensity is a range of variation of a criterion that enables one to distinguish the quality of an alternative for that criterion. An intensity may be expressed as a numerical range of values if the criterion is measurable or in qualitative terms. For example, if ranking students is the objective and one of the criteria on which they are to be ranked is performance in mathematics, the mathematics ratings might be: excellent, good, average, below average, poor; or, using the usual school terminology, A, B, C, D, and F, or a third way is to use numerical ranges, 93-100, 85-95, 75-85, 60-75, below 60. Relative comparisons are first used to set priorities on the ratings themselves. If desired, one can fit a continuous curve through the derived intensities. It is perfectly reasonable to ask how much an A is preferred to a B or to a C. The judgment of how much an A is preferred to a B might be different under different criteria. Perhaps for mathematics an A is very strongly preferred to a B, while for physical education an A is only moderately preferred to a B. So the end result might be that the ratings are scaled differently. For example one could have the following scale values for the ratings:

	Math	Physical Education
A	0.50	0.30
B	0.30	0.30
C	0.15	0.20
D	0.04	0.10
E	0.01	0.10

We now illustrate absolute measurement with an example. A firm evaluates its employees for raises. The criteria are dependability, education, experience, and quality. Each criterion is subdivided into intensities, standards, or subcriteria (Figure 4). The managers set priorities for the criteria by comparing them in pairs. They then pairwise compare the intensities according to priority with respect to their parent criterion "dependability" (as in Table 14) or with respect to a subcriterion if they are using a deeper hierarchy. The priorities of the intensities are divided by the largest intensity for each criterion (second column of priorities in Table 15) to put it in the ideal mode (when the alternatives do not depend on how many and what kind of other alternatives there are), otherwise one does not divide by the largest value but simply normalizes the intensities). The managers answer the question: which intensity is more important and by how much with respect to dependability. The judgments would be different if dependability is applied to waiters in a restaurant than to captains of a passenger airplane. Finally, the managers rate each individual (Table 16) by assigning the intensity rating that applies to him or her under each criterion. The scores of these intensities are each weighted by the priority of its criterion and summed to derive a total ratio scale score for the individual (shown on the right of Table 16). These numbers belong to a ratio scale, and the managers can give salary increases in proportion to the ratios of these numbers after dividing each by the sum of all the rating priorities. Adams gets the highest score and Kesselman the lowest. This approach can be used whenever it is possible to set priorities for intensities of criteria; people can usually do this when they have sufficient experience with a given operation. This normative mode requires that alternatives be rated one by one without regard to how many there may be and how high or low any of them rates on prior standards. Some corporations have insisted that they no longer trust the normative standards of their experts and that they prefer to make paired comparisons of their alternatives. Still, when there is wide agreement on standards, the absolute mode saves time in rating a large number of alternatives. In addition, one can use this approach to rate many alternatives but then choose the top few and perform paired comparisons on them directly with respect to the criteria by deleting the intensities from the hierarchy.

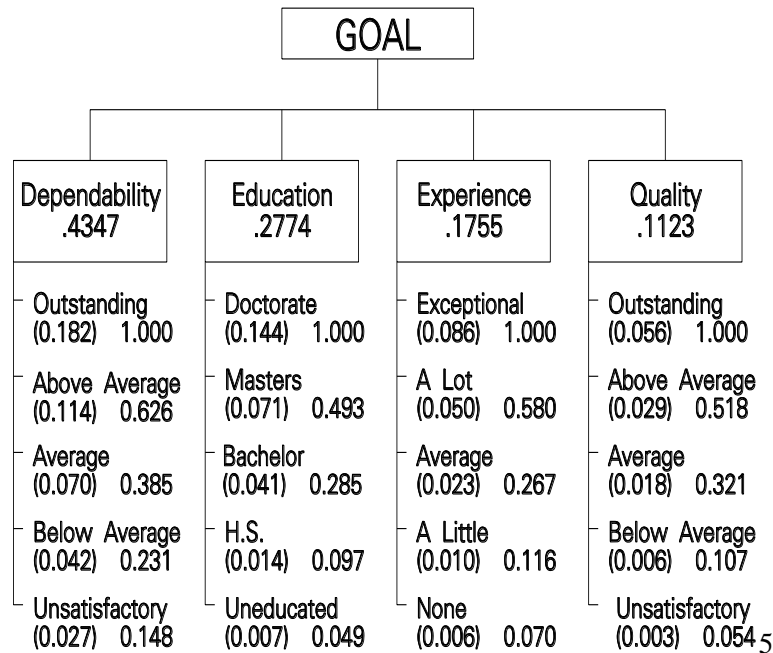


Figure 4. Criteria and Intensities for Rating Employees

Table 14 gives a comparison of the intensities for the dependability criterion. The other intensities are similarly compared.

Table 14. Comparing Intensities of Dependability

	Outstanding	Above Average	Average	Below Average	Unsatisfactory	Priorities
Outstanding	1.0	2.0	3.0	4.0	5.0	0.419
Above Average	1/2	1.0	2.0	3.0	4.0	0.263
Average	1/3	1/2	1.0	2.0	3.0	0.160
Below Average	1/4	1/3	1/2	1.0	2.0	0.097
Unsatisfactory	1/5	1/4	1/3	1/2	1.0	0.062

C.R. = 0.015

Table 15. The Ideal Intensity Mode

	Priorities Weighted by .4347	Divide by largest value
Outstanding	0.182	1.000
Above Average	0.114	0.626
Average	0.070	0.385
Below Average	0.042	0.231
Unsatisfactory	0.027	0.148

Table 16. Ranking Employees against Standards

	Dependability .4347	Education .2774	Experience .1775	Quality .1123	Total
1. Adams, V	Outstanding	Bachelor	A Little	Outstanding	0.646
2. Becker, L	Average	Bachelor	A Little	Outstanding	0.379

3. Hayat, F	Average	Masters	A Lot	Below Average	0.418
4. Kesselman, S	Above Average	H.S.	None	Above Average	0.369
5. O'Shea, K	Average	Doctorate	A Lot	Above Average	0.605
6. Peters, T	Average	Doctorate	A Lot	Average	0.583
7. Tobias, K	Above Average	Bachelor	Average	Above Average	0.456

CAUTION - A REASONABLE LOOKING SCALE MAY BE ORDINAL

It is inadvisable to simply assign numbers mechanically to the intensities of the terminal criteria (just above the alternatives) because we have no proof that for each criterion they actually belong to a ratio scale. For example, to derive the scale 1 to 9 from a consistent matrix whose first row is 1, 1/2, ..., 1/9 and then use the corresponding column vector 1, 2, ..., 9 with uniform spacing may be valid once in a while but cannot be done by fiat. This sort of "slight of mind" scaling has been adopted on occasion by different individuals using the same scale for the intensities. The outcome of the overall rating is questionable. One might call it robotized ranking. Even if one were able to justify the use of such numbers on occasion, it would be no substitute for comparing intensities on the basis of carefully elicited judgments in every case. The AHP requires use of ratio scales for each criterion with careful justification. Otherwise, the scales are more likely to be ordinals. There would then not be a valid reason to use a multicriteria approach.

COMBINING ABSOLUTE AND RELATIVE MEASUREMENT

It is possible to combine absolute and relative measurement in a hierarchy. One may want to use intensities for some of the terminal criteria for rating each alternative. These ratings of the alternatives on the intensities are synthesized and normalized and then combined (added) with the paired comparison outcome of the alternatives with respect to the remaining terminal criteria. The ideal approach (dividing each set of intensities by the largest values) of the absolute mode is used in the ideal case. For more examples which apply the ideal mode and absolute measurement, see the book by Saaty and Vargas (1994): *Decision Making in Economic, Political, Social, and Technological Environments* published by RWS Publications.

SENSITIVITY ANALYSIS

It is often desirable to test the responsiveness or sensitivity of the outcome of a decision to changes in the priorities of the major criteria of that problem. What one does is to change the priority of that criterion keeping the proportions of the priorities for the other criteria the same so again they all, including the changed criterion, add to one. The software Expert Choice has at least five ways to display the result of such sensitivity changes. It is best to illustrate with an example. We have seven criteria and five alternatives for buying a car shown in the hierarchy of Figure 5. The five ways for sensitivity display are shown in Figure 6 followed by an explanation of each.

GOAL (1.000)						
\$INITIAL (0.082)	\$MAINT (0.080)	\$FUEL (0.036)	\$RESALE (0.080)	\$STATUS (0.254)	COMFORT (0.254)	REL'BLTY (0.216)
V.W.	V.W.	V.W.	V.W.	V.W.	V.W.	V.W.
Honda	Honda	Honda	Honda	Honda	Honda	Honda
Chevy	Chevy	Chevy	Chevy	Chevy	Chevy	Chevy
Cadillac	Cadillac	Cadillac	Cadillac	Cadillac	Cadillac	Cadillac

Mercedes	Mercedes	Mercedes	Mercedes	Mercedes	Mercedes	Mercedes
----------	----------	----------	----------	----------	----------	----------

Figure 5. Hierarchy for Buying a Car

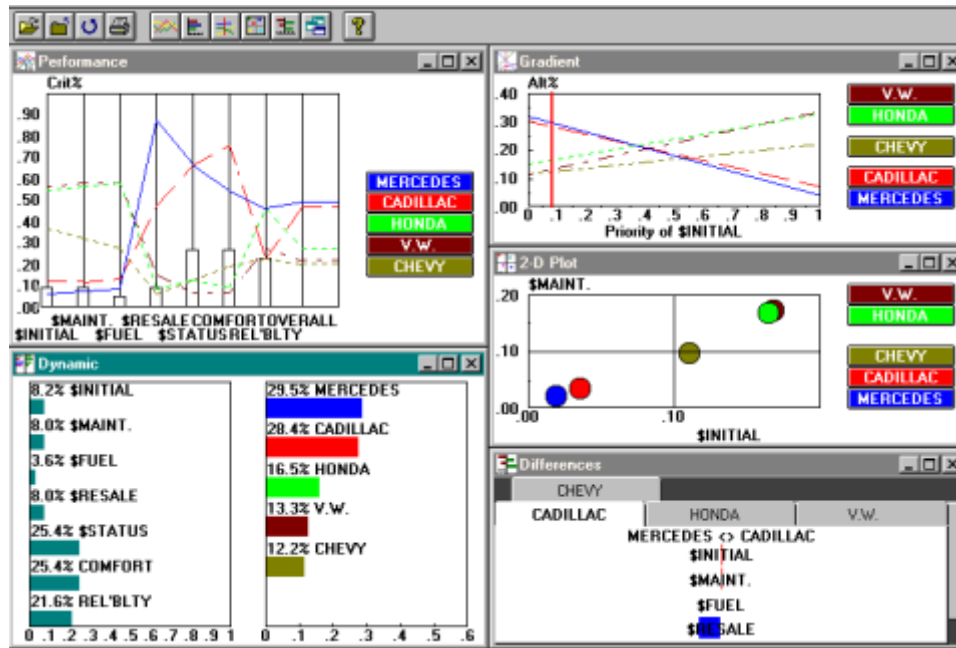


Figure 6. Five Ways to Evaluate Sensitivity

Performance Sensitivity: All information about how alternatives behave on each of the criteria is put in a single graph. Each criterion is represented by a vertical line and the points at which the lines representing the alternatives cross that line indicate the values the alternatives have for that criterion, as measured on the right hand scale. The vertical line next to the right hand scale, labeled overall, shows the composite weight for each alternative as do the intersections with the scale itself. The priority of a criterion is shown by the height of its rectangle as read from the left scale.

Dynamic Sensitivity: Both criteria and alternatives are represented by horizontal bars on the left and on the right respectively. Varying the length of the criteria bars gives rise to appropriate variations in the lengths of the bars representing the priorities of the alternatives. When one criterion bar is moved outward for example the others automatically and proportionately move inward.

Gradient Sensitivity: Shows the variation of the priorities of the alternatives corresponding to variations in the priority of a single criterion. The intersection of the vertical line with the horizontal scale shows the actual value of the criterion as it arises in the problem. The intersection of the alternatives' lines with the left hand vertical scale, represent the alternatives' priorities. Moving this line to the left or to the right shows how the alternatives' priorities change as the priority of the criterion changes.

Two dimensional plot: Shows how well the alternatives (represented by circles) perform with respect to pairs of criteria one on the x-axis and the other on the y-axis. The figure here shows their projection on the diagonal of the rectangle. The farther out on this composite line the projection of a point falls, the better that alternative rates on the two criteria.

Weighted Differences Sensitivity: The lengths of the horizontal bars show the difference for each pair of cars, here given for the Cadillac global priority and the Mercedes global priority, on each of the criteria. If it is positive it is in favor of the Cadillac (represented on the right), if it is negative it is in favor of the Mercedes (represented on the left). The overall difference scale is the bottom line in the figure. All pairs of alternatives may be examined in this way. The words in the left side of the figure are not very clear. They are: Initial, Maintenance, Fuel, Resale, Status, Comfort, and Reliability.

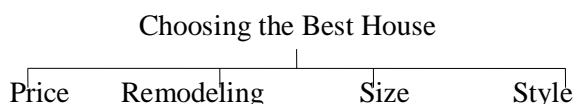
HOMOGENEITY AND CLUSTERING

Think of the following situation: we need to determine the relative size of a blueberry and a watermelon. Here, we need a range greater than 1 to 9. Human beings have difficulty establishing appropriate relationships when the ratios get beyond 9. To resolve this human difficulty, we can use a method in which we cluster different elements so we can rate them within a cluster and then rate them across the clusters. We need to add other fruits to make the comparison possible and then form groups of comparable fruits. In the first group we include the blueberry, a grape, and a plum. In the second group we include the same plum, an apple, and a grapefruit. In the third group we include the same grapefruit, a melon, and the watermelon. The AHP requires reciprocal comparisons of homogeneous elements whose ratios do not differ by much on a property, hence the absolute scale 1—9; when the ratios are larger, one must cluster the elements in different groups and use a common element (pivot) that is the largest in one cluster and the smallest element in the next cluster of the next higher order of magnitude. The weights of the elements in the second group are divided by the priority of the pivot in that group and then multiplied by the priority of the same pivot element (whose value is generally different) from the first group, making the two clusters comparable on the same ratio scale and can now be put in one cluster. The process is then continued to the next cluster and so on. The AHP software program Expert Choice performs these functions for the user. The reason for using clusters of a few elements is to ensure greater stability of the priorities in face of inconsistent judgments. Comparing more than two elements allows for redundancy and hence also for greater validity of real-world information. The AHP often uses seven elements to maintain reasonable consistency in judgments and puts them in clusters. If there are more than seven elements, they are divided into two or more clusters, with each cluster governed by a dummy node that is assigned the sum of the local priorities of the elements in its subcluster. (Elaborate mathematical derivations are given in the AHP to show that the number of elements compared should not be too large in order to obtain priorities with admissible consistency.)

WHAT TO DO WHEN KNOWN SCALES ARE INVOLVED

Sometimes several criteria may occur each of which is measured in units of some known scale and one wants to use those scale values without pairwise comparing their significance. For example, in some cases one may want to use money values without interpreting their importance. The real problem occurs when the same scale, like dollars is used for more than one criterion. When this happens, it is necessary to assign the criteria having the same scale the sum of the measurements of the alternatives under it to the total measurement of the alternatives under each of the criteria that have this measurement.

Assume that a family is considering buying a house and there are three houses to consider. Four factors dominate their thinking: the price of the house, the remodeling costs, the size of the house as reflected by its footage and the style of the house which is an intangible. They have looked at three houses with data shown below on the quantifiables:



	(\$1000)	Costs (\$300)	(sq.ft.)	
A	200	150	3000	Colonial
B	300	50	2000	Ranch
C	500	100	5500	Split Level

First we normalize the values under each of the quantifiable factors obtaining:

Choosing the Best House				
	Price (1000/1300)	Remodeling Costs (300/1300)	Size (sq.ft.)	Style
A	200/1000	150/300	3000/10500	Colonial
B	300/1000	50/300	2000/10500	Ranch
C	500/1000	100/300	5500/10500	Split Level

Then we must combine the factors measured with respect to the same scale in this case, price. We have:

Choosing the Best House		
Economic Factors (combining Price and Remodeling Cost)	Size	Style
A 350/1300	3000/10500	Colonial
B 350/1300	2000/10500	Ranch
C 600/1300	5500/10500	Split Level

Next we establish priorities for the factors through paired comparisons. The resulting priorities are used to weight the normalized values of the alternatives as they are given here. At times in addition to the weight derived from measurement and assigned to the criteria, one may also want to compare and weight the criteria. In that case corresponding values for the two sets of weights can be multiplied and then normalized and used to combine the alternatives under the Economic Factors node.

Alternatively, and in some cases better, we can use the measurements as data helpful to perform paired comparisons. The former approach using the actual measurements would be probably used by a disinterested party for whom the dollar costs are simply numbers of no personal significance, although they might have substantially different significance to the family buying the house. To treat dollars (or any other scale measurements) directly without comparisons can yield misleading results as to the real priority or significance of their actual value. In this example, tangible and intangible factors had to be compared. Their priorities are used to weight the priorities of the alternatives. These priorities are obtained by converting measurements to priorities directly through normalization (seldom justified) or by interpreting their relative importance through judgment (essential where no measurements are possible).

There are two other cases to consider. The first is when gambles and expected values are involved between factors measured on the same scale which are then combined accordingly into a single factor for that scale. The other may involve relations between subfactors from a common scale and intangible factors. The subfactors must be normalized with the other subfactors as above and then combined with

the intangibles accordingly. The other subfactors from the given scale would be combined into a single scale factor as above and one proceeds in the usual way to complete the prioritization process.

TRANSFORMING SCALE READINGS TO PRIORITIES

If we were to make a distinction between measurable attributes such as temperature and the criteria resulting from them such as warmth or comfort, we quickly realize that while we can measure temperature on a thermometer continuously, the impact of temperature readings on our comfort are not as sensitive to variations in the measurements of temperature. For example a temperature of 55 degrees and a temperature of 60 degrees Fahrenheit may be regarded as close, whereas a subfreezing temperature of 30 degrees and 35 degrees may not be thought of as close. To represent temperature variations on a priority scale, we pairwise compare different regions of temperature intensity as to our preference for comfort: Extremely cold, very very cold, freezing, cold, cool, warm, hot, very hot. We can use the resulting scale to draw a continuous curve whose x-axis consists of temperature ranges corresponding to the intensities. We then fill in the scale values in a continuous fashion on the x-axis. In this way we can assign priorities to every value of temperature that is of concern to us and use these priorities as needed in a hierarchic framework.

PROBLEMS WITH ANALYTIC DECISION MAKING

At this point you may wonder why we have three different modes for establishing priorities, the absolute measurement mode and the distributive and ideal modes of relative measurement. Isn't one enough? Let me explain why we need more than one mode.

A major reason for having more than one mode is concerned with this question. What happens to the synthesized ranks of alternatives when new ones are added or old ones deleted? With consistent judgments, the original relative rank order cannot change under any single criterion, but it can under several criteria.

Assume that an individual has expressed preference among a set of alternatives, and that as a result, he or she has developed a ranking for them. Can and should that individual's preferences and the resulting rank order of the alternatives be affected if alternatives are added to the set or deleted from it and if no criteria are added or deleted, which would affect the weights of the old criteria? What if the added alternatives are copies or near copies of one or of several of the original alternatives and their number is large? Rank reversal is an unpleasant property if it is caused by the addition of truly irrelevant alternatives. However, the addition of alternatives may just reflect human nature: the straw that broke the camel's back was considered irrelevant along with all those that went before it. Mathematically, the number and quality of newly added alternatives are known to affect preference among the original alternatives. Most people, unaided by theory and computation, make each decision separately, and they are not very concerned with rank reversal unless they are forced for some reason to refer to their earlier conclusions. It is essential to understand and deal with this phenomenon.

AN EXAMPLE OF RANK REVERSAL

Two products A and B are evaluated according to two equally important attributes P and Q as in the following matrices:

P	A	B	Priorities	Q	A	B	Priorities
---	---	---	------------	---	---	---	------------

A	1	5	.83
B	1/5	1	.17

A	1	1/3	.25
B	3	1	.75

We obtain the following priorities: $W_A = .542$, $W_B = .458$, and A is preferred to B.

A third product C is then introduced and compared with A and B as follows:

P	A	B	C	Priorities
A	1	5	1	.455
B	1/5	1	1/5	.090
C	1	5	1	.455

Q	A	B	C	Priorities
A	1	1/3	2	.222
B	3	1	6	.666
C	1/2	1/6	1	.111

Synthesis yields $W_A = .338$, $W_B = .379$, and $W_C = .283$. Here B is preferred to A and there is rank reversal.

For a decision theory to have a lasting value, it must consider how people make decisions naturally and assist them in organizing their thinking to improve their decisions in that natural direction. Its assumptions should be tied to evolution and not to present day determinism. This is the fundamental concept on which the AHP is based. It was developed as a result of a decade of unsuccessful attempts to use normative theories, with the assistance of some of the world's best minds, to deal with negotiation and trade-off in the strategic political and diplomatic arena at the Arms Control and Disarmament Agency in the Department of State. In the early 1970s, I asked the question, how do ordinary people process information in their minds in attempting to make a decision and how do they express the strength of their judgments? The answer to this question led me to consider hierarchies and networks, paired comparisons, ratio scales, homogeneity and consistency, priorities, ranking, and the AHP.

HOW CLOSE ARE TWO RATIO SCALE READINGS?

If we are given two sets of readings from the same ratio scale, how do we determine if they are close or not? We form the matrix of all ratios from the first set of readings and the transpose around the diagonal of the matrix of all ratios from the second set of readings, multiply corresponding elements, add and divide by n^2 . If the result is close to 1.10 the ratios may be taken to be close, otherwise not. The reason for this is that if the two ratio scales are the same, then their ratios would be the same and if we divide the entries of ratios of one matrix by the corresponding entries from the other matrix we would get a one in every position. Instead of dividing we can multiply by the reciprocals of the elements in the second matrix. That would be the same as multiplying the entries in the first matrix one by one by the corresponding entries in which the columns of the second matrix become the rows of another matrix known as the transpose of the second matrix. If all the ratios (or now products) are equal to one, the two ratio scales are obviously the same. Otherwise, we add the elements of the resulting product matrix and their sum would always exceed the number of entries n^2 . The reason for this is that we are adding pairs of elements of the form $x + 1/x$ whose sum is always not less than two. Thus if we divide by n^2 we want the result to be no more than 1.10. Here are three examples.

Are the ratio scale vectors and close? First we form the matrices of ratios: A from the first scale and B^T from the second and multiply their corresponding elements to obtain:

On adding the elements of the matrix on the right and dividing by $n^2=9$ we obtain:

$$(1/n^2) e^T A \circ B^T e = (9 \frac{1}{4}) / 9 = 1.028$$

Since this value is less than 1.10 we consider the two vectors to be close.

The closeness of the ratio scale vectors $\left(\frac{4}{7}, \frac{2}{7}, \frac{1}{7}\right)^T$ and $\left(\frac{9}{13}, \frac{3}{13}, \frac{1}{13}\right)^T$ may be considered as a borderline case. We have:

from which we obtain

$$\frac{1}{n^2} 1e^T A \circ B^T e = \frac{10 \frac{1}{36}}{9} = 1.114$$

The ratio scale vectors by any measure are not close. Thus,

from which we have

$$\frac{1}{n^2} 2e^T A \circ B^T e = \frac{30.027}{9} = 3.336$$

PERSPECTIVE

From the beginning of the chapter to this point are summarized many of the concepts of the AHP that will arise in doing problems. Benefit and cost hierarchies were discussed, and also benefit, cost, and risk hierarchies. In problems later in the book often a fourth hierarchy of opportunities is included as well. When there is dependence among the alternatives according to number and quality we discussed using the distributive or the ideal mode depending on whether you want to allow rank reversal or prevent it when adding or deleting alternatives. Absolute measurement was discussed in which intensity priority scales are developed for each criterion and the alternatives are rated one by one on these intensities to obtain an overall ranking. This ranking need not be the same as a ranking obtained by pairwise comparing the alternatives. A discussion was given of the need to arrange the elements of a paired comparison into homogeneous clusters linked by a common element to facilitate combining their priorities. Another subject that was discussed is how to establish priorities for criteria whose intensities are given as absolute measurements such as dollars but when in addition the criteria may also have separately established priorities. A discussion was given about how to associate priorities with values from a scale of measurement. Finally a ratio scale metric was described that can be used to compare two separate ratio scale readings to determine how close they are.

KEY CONCEPTS

- ☐ The distributive and ideal modes for synthesizing the priorities of the alternatives help one distinguish between the influence of the number and quality of new alternatives on old ones in ranking them.
- ☐ Dependence on the number of alternatives and on the quality of the alternatives considered -- rank preservation and reversal are part of the decision itself and require two modes to perform the synthesis.
- ☐ Absolute measurement and the use of intensities to rate rather than compare alternatives and still derive a ratio scale ranking for them depends on expert knowledge.
- ☐ Combining clusters of homogeneous elements is an integral part of the AHP.
- ☐ If some criteria have actual measurement, their weights derive from their measurement but if desired their weights can be combined with priorities obtained from paired comparisons.
- ☐ The ratio scale metric can be used to determine how close two sets of priorities are to each other.

4. MATHEMATICS OF THE ANALYTIC HIERARCHY PROCESS

The AHP has four axioms, (1) reciprocal judgments, (2) homogeneous elements, (3) hierarchic or feedback dependent structure, and (4) rank order expectations.

Assume that one is given n stones, A_1, \dots, A_n , with known weights w_1, \dots, w_n , respectively, and suppose that a matrix of pairwise ratios is formed whose rows give the ratios of the weights of each stone with respect to all others. Thus one has the equation:

where \mathbf{A} has been multiplied on the right by the vector of weights \mathbf{w} . The result of this multiplication is $n\mathbf{w}$. Thus, to recover the scale from the matrix of ratios, one must solve the problem $\mathbf{A}\mathbf{w} = n\mathbf{w}$ or $(\mathbf{A} - n\mathbf{I})\mathbf{w} = \mathbf{0}$. This is a system of homogeneous linear equations. It has a nontrivial solution if and only if the determinant of $\mathbf{A} - n\mathbf{I}$ vanishes, that is, n is an eigenvalue of \mathbf{A} . Now \mathbf{A} has unit rank since every row is a constant multiple of the first row. Thus all its eigenvalues except one are zero. The sum of the eigenvalues of a matrix is equal to its trace, the sum of its diagonal elements, and in this case the trace of \mathbf{A} is equal to n . Thus n is an eigenvalue of \mathbf{A} , and one has a nontrivial solution. The solution consists of positive entries and is unique to within a multiplicative constant.

To make \mathbf{w} unique, one can normalize its entries by dividing by their sum. Thus, given the comparison matrix, one can recover the scale. In this case, the solution is any column of \mathbf{A} normalized. Notice that in \mathbf{A} the reciprocal property $a_{ji} = 1/a_{ij}$ holds; thus, also $a_{ii} = 1$. Another property of \mathbf{A} is that it is consistent: its entries satisfy the condition $a_{jk} = a_{ik}/a_{ij}$. Thus the entire matrix can be constructed from a set of n elements which form a chain across the rows and columns.

In the general case, the precise value of w_i/w_j cannot be given, but instead only an estimate of it as a judgment. For the moment, consider an estimate of these values by an expert who is assumed to make small perturbations of the coefficients. This implies small perturbations of the eigenvalues. The problem now becomes $\mathbf{A}'\mathbf{w}' = \lambda_{\max}\mathbf{w}'$ where λ_{\max} is the largest eigenvalue of \mathbf{A}' . To simplify the notation, we shall continue to write $\mathbf{A}\mathbf{w} = \lambda_{\max}\mathbf{w}$, where \mathbf{A} is the matrix of pairwise comparisons. The problem now is how good is the estimate of \mathbf{w} . Notice that if \mathbf{w} is obtained by solving this problem, the matrix whose entries are w_i/w_j is a consistent matrix. It is a consistent estimate of the matrix \mathbf{A} . \mathbf{A} itself need not be consistent. In fact, the entries of \mathbf{A} need not even be transitive; that is, A_1 may be preferred to A_2 and A_2 to A_3 but A_3 may be preferred to A_1 . What we would like is a measure of the error due to inconsistency. It turns out that \mathbf{A} is consistent if and only if $\lambda_{\max} = n$ and that we always have $\lambda_{\max} \geq n$.

Since small changes in a_{ij} imply a small change in λ_{\max} , the deviation of the latter from n is a deviation from consistency and can be represented by $(\lambda_{\max} - n)/(n-1)$, which is called the *consistency index (C.I.)*. When the consistency has been calculated, the result is compared with those of the same index of a randomly generated reciprocal matrix from the scale 1 to 9, with

reciprocals forced. This index is called the *random index (R.I.)*. The following gives the order of the matrix (first row) and the average R.I. (second row):

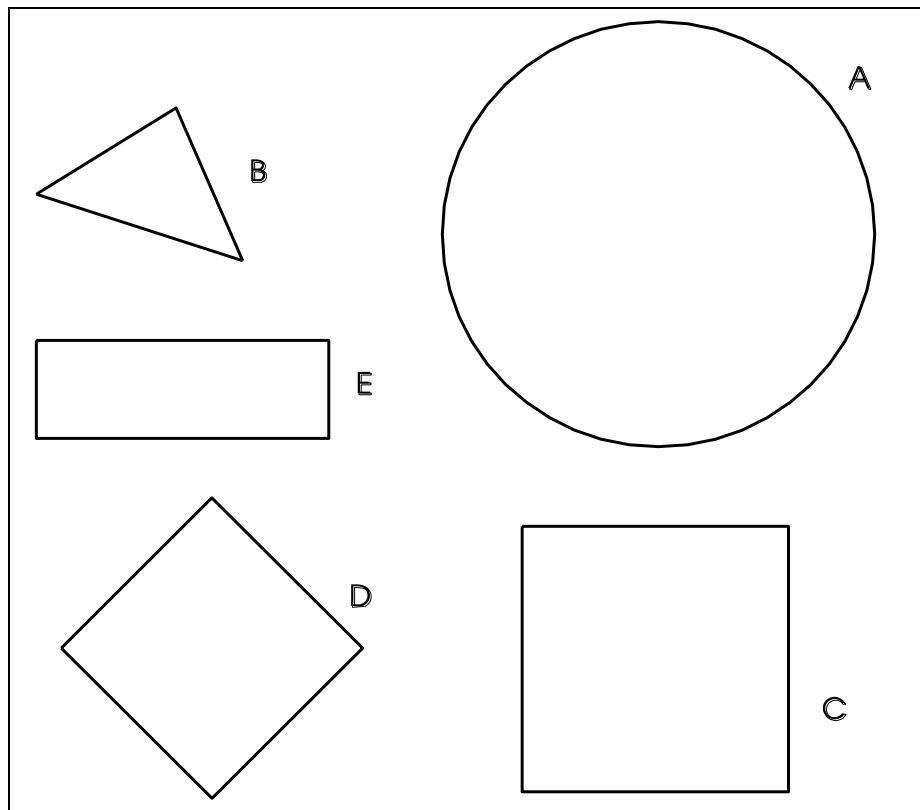
n	1	2	3	4	5	6	7	8	9	10
Random Consistency Index (R.I.)	0	0	.52	.89	1.11	1.25	1.35	1.40	1.45	1.49

The ratio of C.I. to the average R.I. for the same order matrix is called the *consistency ratio (C.R.)*. A consistency ratio of 0.10 or less is positive evidence for informed judgment.

The relations $a_{ji} = 1/a_{ij}$ and $a_{ii} = 1$ are preserved in these matrices to improve consistency. The reason for this is that if stone #1 is estimated to be k times heavier than stone #2, one should require that stone #2 be estimated to be $1/k$ times the weight of the first. If the consistency ratio is significantly small, the estimates are accepted; otherwise, an attempt is made to improve consistency by obtaining additional information. What contributes to the consistency of a judgment are (1) the homogeneity of the elements in a group, that is, not comparing a grain of sand with a mountain; (2) the sparseness of elements in the group, because an individual cannot hold in mind simultaneously the relations of many more than a few objects; and (3) the knowledge and care of the decision maker about the problem under study.

The drawing below shows five areas to which we can apply to the paired comparison process in a matrix and use the 1 to 9 scale to test the validity of the procedure. We can approximate the priorities in the matrix by assuming that it is consistent. We normalize each column and then take the average of the corresponding entries in the columns.

VALIDATION EXERCISE



The shapes above are drawn to scale. Compare them in pairs, using the fundamental scale of AHP. The object of the validation exercise is to determine the relative areas of the figures expressed as a priority vector. This priority vector resulting from the pairwise comparisons matrix should give the same result as normalizing the actual areas of the figures. Normalizing means divided the area of each figure by the sum of the areas of all the figures. The resulting priority vector should represent their relative areas, that is, if A's priority is divided by B's priority, the result should reveal how many times bigger A is than B.

The actual relative values of these areas are $A=0.47$, $B=0.05$, $C=0.24$, $D=0.14$, and $E=0.09$ with which the answer may be compared. Note that A is about two times as big as C, and about 9 times as big as B. By comparing more than two alternatives in a decision problem, one is able to obtain better values for the derived scale because of redundancy in the comparisons, which helps improve the overall accuracy of the judgments.

5. THE SEVEN PILLARS OF THE ANALYTIC HIERARCHY PROCESS

This section was originally a paper delivered by the author, Thomas L. Saaty, to the International Society of the Analytic Hierarchy Process (ISAHP) meeting in Kobe, Japan, in 1999. The seven pillars of the

AHP, some highlights of which are discussed in the paper, are that: 1) ratio scales derived from reciprocal paired comparisons; 2) paired comparisons and the psychophysical origin of the fundamental scale used to make the comparisons; 3) conditions for sensitivity of the eigenvector to changes in judgments; 4) homogeneity and clustering to extend the scale from 1-9 to 1- ∞ ; 5) additive synthesis of priorities, leading to a vector of multi-linear forms as applied within the decision structure of a hierarchy or the more general feedback network to reduce multi-dimensional measurements to a unidimensional ratio scale; 6) allowing rank preservation (ideal mode) or allowing rank reversal (distributive mode); and 7) group decision making using a mathematically justifiable way for synthesizing individual judgments which allows the construction of a cardinal group decision compatible with the individual preferences.

1. Introduction

The Analytic Hierarchy Process (AHP) provides the objective mathematics to process the inescapably subjective and personal preferences of an individual or a group in making a decision. With the AHP and its generalization, the Analytic Network Process (ANP), one constructs hierarchies or feedback networks, then makes judgments or performs measurements on pairs of elements with respect to a controlling element to derive ratio scales that are then synthesized throughout the structure to select the best alternative.

Fundamentally, the AHP works by developing priorities for alternatives and the criteria used to judge the alternatives. Usually the criteria, whose choice is at the mercy of the understanding of the decision-maker (irrelevant criteria are those that are not included in the hierarchy), are measured on different scales, such as weight and length, or are even intangible for which no scales yet exist. Measurements on different scales, of course, cannot be directly combined. First, priorities are derived for the criteria in terms of their importance to achieve the goal, then priorities are derived for the performance of the alternatives on each criterion. These priorities are derived based on pairwise assessments using judgment, or ratios of measurements from a scale if one exists. The process of prioritization solves the problem of having to deal with different types of scales, by interpreting their significance to the values of the user or users. Finally, a weighting and adding process is used to obtain overall priorities for the alternatives as to how they contribute to the goal. This weighting and adding parallels what one would have done arithmetically prior to the AHP to combine alternatives measured under several criteria having the *same* scale (a scale that is often common to several criteria is money) to obtain an overall result. With the AHP a multidimensional scaling problem is thus transformed to a unidimensional scaling problem.

The seven pillars of the AHP are that: 1) **Ratio scales, proportionality, and normalized ratio scales** are central to the generation and synthesis of priorities, whether in the AHP or in any multicriteria method that needs to integrate existing ratio scale measurements with its own derived scales; in addition, ratio scales are the only way to generalize a decision theory to the case of dependence and feedback because ratio scales can be both multiplied, and added - when they belong to the same scale such as a priority scale; when two judges arrive at two different ratio scales for the same problem one needs to test the compatibility of their answers and accept or reject their closeness. The AHP has a non-statistical index for doing this. Ratio scales can also be used to make decisions within an even more general framework involving several hierarchies for benefits, costs, opportunities and risks, and using a common criterion such as *economic* to ensure commensurability; ratio scales are essential in proportionate resource allocation as in linear programming, recently generalized to deal with relative measurement for both the objective function and the constraints obtaining a ratio scale solution vector form which it is possible to decide on the relative values of the allocated resources; one can associate with each alternative a vector of benefits, costs, time of completion, etc., to determine the best alternative subject to all these general concerns; 2) **Reciprocal paired comparisons** are used to express judgments semantically automatically linking them to a numerical fundamental scale of absolute numbers (derived from stimulus-response relations) from which the principal eigenvector of priorities is then derived; the eigenvector shows the

dominance of each element with respect to the other elements; an element that does not have a particular property is automatically assigned the value zero in the eigenvector without including it in the comparisons; dominance along all possible paths is obtained by raising the matrix to powers and normalizing the sum of the rows; inconsistency in judgment is allowed and a measure for it is provided which can direct the decision maker in both improving judgment and arriving at a better understanding of the problem; scientific procedures for giving less than the full set of $n(n-1)/2$ judgments in a matrix have been developed; using interval judgments eventually leading to the use of optimization and statistical procedures is a complex process which is often replaced by comparing ranges of values of the criteria, performing sensitivity analysis, and relying on conditions for the insensitivity of the eigenvector to perturbations in the judgments; the judgments may be considered as random variables with probability distributions; the AHP has at least three modes for arriving at a ranking of the alternatives: a) **Relative**, which *rank*s a few alternatives by comparing them in pairs and is particularly useful in new and exploratory decisions, b) **Absolute**, which *rate*s an unlimited number of alternatives one at a time on intensity scales constructed separately for each covering criterion and is particularly useful in decisions where there is considerable knowledge to judge the relative importance of the intensities and develop priorities for them; if desired, a few of the top rated alternatives can then be compared against each other using the relative mode to obtain further refinement of the priorities; c) **Benchmarking**, which ranks alternatives by including a known alternative in the group and comparing the other against it; 3) **Sensitivity of the principal right eigenvector** to perturbation in judgments limits the number of elements in each set of comparisons to a few and requires that they be homogeneous; the left eigenvector is only meaningful as reciprocal; due to the choice of a unit as one of the two elements in each paired comparison to determine the relative dominance of the second element, it is not possible to derive the principal left eigenvector directly from paired comparisons as the dominant element cannot be decomposed a priori; as a result, to ask for how much less one element is than another we must take the reciprocal of what we get by asking how much more the larger element is; 4) **Homogeneity and clustering** are used to extend the fundamental scale gradually from cluster to adjacent cluster, eventually enlarging the scale from 1-9 to 1-infinity; 5) **Synthesis that can be extended to dependence and feedback** is applied to the derived ratio scales to create a uni-dimensional ratio scale for representing the overall outcome. Synthesis of the scales derived in the decision structure can only be made to yield correct outcomes on known scales by additive weighting. It should be carefully noted that additive weighting in a hierarchical structure leads to a multilinear form and hence is nonlinear. It is known that under very general conditions such multilinear forms are dense in general function spaces (discrete or continuous), and thus linear combinations of them can be used to approximate arbitrarily close to any nonlinear element in that space. Multiplicative weighting, by raising the priorities of the alternatives to the power of the priorities of the criteria (which it determines through additive weighting!) then multiplying the results, has four major flaws: a) It does not give back weights of existing same ratio scale measurements on several criteria as it should; b) It assumes that the matrix of judgments is always consistent, thus sacrificing the idea of inconsistency and how to deal with it, and not allowing redundancy of judgments to improve validity about the real world; c) Most critically, it does not generalize to the case of interdependence and feedback, as the AHP generalizes to the Analytic Network Process (ANP), so essential for the many decision problems in which the criteria and alternatives depend on each other; d) It always preserves rank which leads to unreasonable outcomes and contradicts the many counterexamples that show rank reversals should be allowed; 6) **Rank preservation and reversal** can be shown to occur without adding or deleting criteria, such as by simply introducing enough copies of an alternative or for numerous other reasons; this leaves no doubt that rank reversal is as intrinsic to decision making as rank preservation also is; it follows that any decision theory must have at least two modes of synthesis; in the AHP they are called the distributive and ideal modes, with guidelines for which mode to use; rank can always be preserved by using the ideal mode in both absolute measurement and relative measurement; 7) **Group judgments** must be integrated one at a time carefully and mathematically, taking into consideration when desired the experience, knowledge, and power of each person involved in the decision, without the need to force consensus, or to use majority or other ordinal ways of voting; the

theorem regarding the *impossibility* of constructing a social utility function from individual utilities that satisfies four reasonable conditions which found their validity with *ordinal* preferences is *no longer true* when *cardinal* ratio scale preferences are used as in the AHP. Instead, one has the *possibility* of constructing such a function. To deal with a large group requires the use of questionnaires and statistical procedures for large samples.

2. Ratio Scales

A *ratio* is the relative value or quotient a/b of two quantities a and b of the same kind; it is called commensurate if it is a rational number, otherwise it is incommensurate. A statement of the equality of two ratios a/b and c/d is called *proportionality*. A ratio scale is a set of numbers that is invariant under a similarity transformation (multiplication by a positive constant). The constant cancels when the ratio of any two numbers is formed. Either pounds or kilograms can be used to measure weight, but the ratio of the weight of two objects is the same for both scales. An extension of this idea is that the weights of an entire set of objects whether in pounds or in kilograms can be standardized to read the same by normalizing. In general if the readings from a ratio scale are aw_i^* , $i=1, \dots, n$, the standard form is given by $w_i = aw_i^* / aw_i^* = w_i^* / w_i^*$ as a result of which we have $w_i = 1$, and the w_i , $i=1, \dots, n$, are said to be normalized. We no longer need to specify whether weight for example is given in pounds or in kilograms or in another kind of unit. The weights (2.21, 4.42) in pounds and (1, 2) in kilograms, are both given by (1/3, 2/3) in the standard ratio scale form.

The relative ratio scale derived from a pairwise comparison reciprocal matrix of judgments is derived by solving:

$$\sum_{j=1}^n a_{ij} w_j = \lambda_{\max} w_i \quad (1) \quad \sum_{i=1}^n w_i = 1 \quad (2)$$

with $a_{ji}=1/a_{ij}$ or $a_{ij} a_{ji}=1$ (the reciprocal property), $a_{ij} > 0$ (thus A is known as a positive matrix) whose solution, known as the principal right eigenvector, is normalized as in (2). A relative ratio scale does not need a unit of measurement.

When $a_{ij} a_{jk} = a_{ik}$, the matrix $A=(a_{ij})$ is said to be consistent and its principal eigenvalue is equal to n . Otherwise, it is simply reciprocal. The general eigenvalue formulation given in (1) is obtained by perturbation of the following consistent formulation:

$$A w = \begin{bmatrix} A_1 & \dots & A_n \\ \vdots & & \vdots \\ A_n & \dots & A_n \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} n$$

where A has been multiplied on the right by the transpose of the vector of weights $w = (w_1, \dots, w_n)$. The result of this multiplication is nw . Thus, to recover the scale from the matrix of ratios, one must solve the problem $Aw = nw$ or $(A - nI)w = 0$. This is a system of homogeneous linear equations. It has a nontrivial solution if and only if the determinant of $A - nI$ vanishes, that is, n is an eigenvalue of A . Now A has unit rank since every row is a constant multiple of the first row. Thus all its eigenvalues except one are zero. The sum of the eigenvalues of a matrix is equal to its trace, that is, the sum of its diagonal elements. In

this case the trace of A is equal to n . Thus n is an eigenvalue of A , and one has a nontrivial solution. The solution consists of positive entries and is unique to within a multiplicative constant.

The discrete formulation given in (1) and (2) above generalizes to the continuous case through Fredholm's integral equation of the second kind and is given by:

$$\int_a^b K(s,t) w(t) dt = \lambda_{\max} w(s) \quad (3); \quad \lambda \int_a^b K(s,t) w(t) dt = w(s) \quad (4); \quad \int_a^b w(s) ds = 1 \quad (5)$$

where instead of the matrix A we have as a positive kernel, $K(s,t) > 0$. Note that the entries in a matrix depend on the two variables i and j which assume discrete values. Thus the matrix itself depends on these discrete variables, and its generalization, the kernel function also depends on two (continuous) variables. The reason for calling it kernel is the role it plays in the integral, where without knowing it we cannot determine the exact form of the solution. The standard way in which (3) is written is to move the eigenvalue to the left hand side which gives it the reciprocal form. In general, by abuse of notation, one continues to use the symbol λ to represent the reciprocal value. Our equation for response to a stimulus is now written in the standard form (4) with the normalization condition (5). Here also, we have the reciprocal property (6) and as in the finite case, the kernel $K(s,t)$ is consistent if it satisfies the relation (7):

$$K(s,t) K(t,s) = 1 \quad (6); \quad K(s,t) K(t,u) = K(s,u), \text{ for all } s, t, \text{ and } u \quad (7)$$

An example of this type of kernel is $K(s,t) = e^{s-t} = e^s / e^t$. It follows by putting $s=t=u$, that $K(s,s)=1$ for all s which is analogous to having ones down the diagonal of the matrix in the discrete case. A value of λ for which Fredholm's equation has a nonzero solution $w(t)$ is called a characteristic value (or its reciprocal is called an eigenvalue) and the corresponding solution is called an eigenfunction. An eigenfunction is determined to within a multiplicative constant. If $w(t)$ is an eigenfunction corresponding to the characteristic value λ and if C is an arbitrary constant, we can easily see by substituting in the equation that $Cw(t)$ is also an eigenfunction corresponding to the same λ . The value $\lambda=0$ is not a characteristic value because we have the corresponding solution $w(t)=0$ for every value of t , which is the trivial case, excluded in our discussion.

It may be useful to recount a little of the history of how Fredholm's equation came about in the ratio scale formulation of the AHP. My student Hasan Ait-Kaci and I first recognized the connection between Fredholm's equation and the AHP in a paper we wrote in the late 1970's. In the early 1980's, I and my friend and colleague, Professor Luis Vargas, used this formulation in the framework of neural firing and published several papers on the subject. In December of 1996, I had the nagging idea that the ratio scale relation for electrical firing was not reflected in our solution, and that periodicity had to be involved in the solution with which I began. Many researchers on the brain had considered neural firing in the framework of a damped periodic oscillator. It was my friend Janos Aczel, the leading functional equation mathematician in the world, who provided me with a variety of solutions for the functional equation ($w(as) = bw(s)$). I had proved in the theorem given below that this equation characterizes the solution of Fredholm's equation and its solution is an eigenfunction of that equation. My work is an extension of the work I had done earlier with Vargas. The solution has the form of a damped periodic oscillator of period one. It has an additional logarithmic property that corresponds to Fechner's law discussed later in this paper.

A matrix is consistent if and only if it has the form $A=(w_i / w_j)$ which is equivalent to multiplying a column vector that is the transpose of (w_1, \dots, w_n) by the row vector $(1/w_1, \dots, 1/w_n)$. As we see below, the kernel $K(s,t)$ is separable and can be written as

$$K(s,t) = k_1(s) k_2(t)$$

Theorem $K(s,t)$ is consistent if and only if it is separable of the form:

$$K(s,t)=k(s)/k(t) \quad (8)$$

Theorem If $K(s,t)$ is consistent, the solution of (4) is $gk(s)$ by $w(s) = \frac{k(s)}{\int_s k(s)ds}$

(9)

In the discrete case, the normalized eigenvector was independent of whether all the elements of the pairwise comparison matrix A are multiplied by the same constant a or not, and thus we can replace A by aA and obtain the same eigenvector. Generalizing this result we have:

$$K(as, at)=aK(s,t)=k(as)/k(at)=a k(s)/k(t)$$

which means that K is a homogeneous function of order one. In general, when $f(ax_1, ..., ax_n)=a^n f(x_1, ..., x_n)$ holds, f is said to be homogeneous of order n . Because K is a degenerate kernel, we can replace $k(s)$ above by $k(as)$ and obtain $w(as)$. We have now derived from considerations of ratio scales the following condition to be satisfied by a ratio scale:

Theorem A necessary and sufficient condition for $w(s)$ to be an eigenfunction solution of Fredholm's equation of the second kind, with a consistent kernel that is homogeneous of order one, is that it satisfy the functional equation

$$w(as)=bw(s)$$

where $b=aa$.

We have for the general damped periodic response function $w(s)$,

$$w(s) = Ce^{\log b \frac{\log s}{\log a}} P\left(\frac{\log s}{\log a}\right)$$

where P is periodic of period 1 and $P(0)=1$.

We can write this solution as

$$v(u)=C_1 e^{-\beta u} P(u)$$

where $P(u)$ is periodic of period 1, $u=\log s/\log a$ and $\log ab/-\beta$, $\beta>0$. It is interesting to observe the logarithmic function appear as part of the solution. It gives greater confirmation to the Weber-Fechner law developed in the next section.

3. Paired Comparisons and the Fundamental Scale

Instead of assigning two numbers w_i and w_j and forming the ratio w_i/w_j we assign a single number drawn from the fundamental 1-9 scale of absolute numbers to represent the ratio $(w_i/w_j)/1$. It is a nearest integer approximation to the ratio w_i/w_j . The derived scale will reveal what the w_i and w_j are. This is a central fact about the relative measurement approach of the AHP and the need for a fundamental scale.

In 1846 Weber found, for example, that people while holding in their hand different weights, could distinguish between a weight of 20 g and a weight of 21 g, but could not if the second weight is only 20.5 g. On the other hand, while they could not distinguish between 40 g and 41 g, they could between 40g and 42g, and so on at higher levels. We need to increase a stimulus s by a minimum amount Δs to reach a point where our senses can first discriminate between s and $s + \Delta s$. Δs is called the just noticeable difference (jnd). The ratio $r = \Delta s/s$ does not depend on s . Weber's law states that change in sensation is noticed when the stimulus is increased by a constant percentage of the stimulus itself. This law holds in ranges where Δs is small when compared with s , and hence in practice it fails to hold when s is either too small or too large. Aggregating or decomposing stimuli as needed into clusters or hierarchy levels is an effective way for extending the uses of this law.

In 1860 Fechner considered a sequence of just noticeable increasing stimuli. He denotes the first one by s_0 . The next just noticeable stimulus is given by

$$s_1 = s_0 + \Delta s_0 = s_0 + \frac{\Delta s_0}{s_0} s_0 = s_0(1 + r)$$

based on Weber's law.

Similarly $s_2 = s_1 + \Delta s_1 = s_1(1 + r) = s_0(1 + r)^2 \equiv s_0 \alpha^2$. In general
 $s_n = s_{n-1} \alpha = s_0 \alpha^n$ ($n = 0, 1, 2, \dots$).

Thus stimuli of noticeable differences follow sequentially in a geometric progression. Fechner noted that the corresponding sensations should follow each other in an arithmetic sequence at the discrete points at which just noticeable differences occur. But the latter are obtained when we solve for n . We have

$$n = \frac{\log s_n - \log s_0}{\log \alpha}$$

and sensation is a linear function of the logarithm of the stimulus. Thus if M denotes the sensation and s the stimulus, the psychophysical law of Weber-Fechner is given by

$$M = a \log s + b, \quad a \neq 0$$

We assume that the stimuli arise in making pairwise comparisons of relatively comparable activities. We are interested in responses whose numerical values are in the form of ratios. Thus $b = 0$, from which we must have $\log s_0 = 0$ or $s_0 = 1$, which is possible by calibrating a unit stimulus. Here the unit stimulus is s_0 . The next noticeable stimulus is $s_1 = s_0 \alpha = \alpha$ which yields the second noticeable response $a \log \alpha$. The third noticeable stimulus is $s_2 = s_0 \alpha^2$ which yields a response of $2a \log \alpha$. Thus we have for the different responses:

$$M_0 = a \log s_0, M_1 = a \log \alpha, M_2 = 2a \log \alpha, \dots, M_n = na \log \alpha.$$

While the noticeable ratio stimulus increases geometrically, the response to that stimulus increases arithmetically. Note that $M_0 = 0$ and there is no response. By dividing each M_i by M_1 we obtain the sequence of absolute numbers 1, 2, 3, ... of the fundamental 1-9 scale. Paired comparisons are made by identifying the less dominant of two elements and using it as the unit of measurement. One then determines, using the scale 1-9 or its verbal equivalent, how many times more the dominant member of the pair is than this unit. In making paired comparisons, we use the nearest integer approximation from the scale, relying on the insensitivity of the eigenvector to small perturbations (discussed below). The reciprocal value is then automatically used for the comparison of the less dominant element with the more

dominant one. Despite the foregoing derivation of the scale in the form of integers, someone might think that other scale values would be better, for example using 1.3 in the place of 2. Imagine comparing the magnitude of two people with respect to the magnitude of one person and using 1.3 for how many there are instead of 2.

We note that there may be elements that are closer than 2 on the 1-9 scale, and we need a variant of the foregoing. Among the elements that are close, we select the smallest. Observe the incremental increases between that smallest one and the rest of the elements in the close group. We now consider these increments to be new elements and pairwise compare them on the scale 1-9. If two of the increments are themselves closer than 2 we treat them as identical, assigning a 1 (we could carry this on ad infinitum – but we will not). In the end each component of the eigenvector of comparisons of the increments is added to unity to yield the un-normalized priorities of the close elements for that criterion. Note that only the least of these close elements is used in comparisons with the other elements that can be compared directly using the normal 1-9 scale. Its priority is used to multiply the priorities of these close elements and finally the priorities of all the elements are re-normalized.

How large should the upper value of the scale be? Qualitatively, people have a capacity to divide their response to stimuli into three categories: high, medium and low. They also have the capacity to refine this division by further subdividing each of these intensities of responses into high, medium and low, thus yielding in all nine subdivisions. It turns out, from the requirement of homogeneity developed below, that to maintain stability, our minds work with a few elements at a time. Using a large number of elements in one matrix leads to greater inconsistency.

4. Sensitivity of the Principal Eigenvector Places a Limit on the Number of Elements and Their Homogeneity

To a first order approximation, perturbation Δw_1 in the principal eigenvector w_1 due to a perturbation ΔA in the matrix A where A is consistent is given by:

$$\Delta w_1 = \sum_{j=2}^n (v_j^T \Delta A w_1 / (\lambda_1 - \lambda_j) v_j^T w_j) w_j$$

The eigenvector w_1 is insensitive to perturbation in A , if the principal eigenvalue λ_1 is separated from the other eigenvalues λ_j , here assumed to be distinct, and none of the products $v_j^T w_j$ of left and right eigenvectors is small. We should recall that the nonprincipal eigenvectors need not be positive in all components, and they may be complex. One can show that all the $v_j^T w_j$ are of the same order, and that $v_1^T w_1$, the product of the normalized left and right principal eigenvectors is equal to n . If n is relatively small and the elements being compared are homogeneous, none of the components of w_1 is arbitrarily small and correspondingly, none of the components of v_1^T is arbitrarily small. Their product cannot be arbitrarily small, and thus w is insensitive to small perturbations of the consistent matrix A . The conclusion is that n must be small, and one must compare homogeneous elements. Later we discuss placing a limit on the value of n .

5. Clustering and Using Pivots to Extend the Scale from 1-9 to 1- ∞

In Figure 1, an unripe cherry tomato is eventually and indirectly compared with a large watermelon by first comparing it with a small tomato and a lime, the lime is then used again in a second cluster with a grapefruit and a honey dew where we then divide by the weight of the lime and then multiply by its

weight in the first cluster, and then use the honey dew again in a third cluster and so on. In the end we have a comparison of the unripe cherry tomato with the large watermelon and would accordingly extended the scale from 1-9 to 1-721.

Such clustering is essential, and must be done separately for each criterion. We should note that in most decision problems, there may be one or two levels of clusters and conceivably it may go up to three or four adjacent ranges of homogeneous elements (Maslow put them in seven groupings). Very roughly we have in decreasing order of importance: 1) Survival, health, family, friends and basic religious beliefs some people were known to die for; 2) Career, education, productivity and lifestyle; 3) Political and social beliefs and contributions; 4) Beliefs, ideas, and things that are flexible and it does not matter exactly how one advocates or uses them. Nevertheless one needs them, such as learning to eat with a fork or a chopstick or with the fingers as many people do interchangeably. These categories can be generalized to a group, a corporation, or a government. For very important decisions, two categories may need to be considered. Note that the priorities in two adjacent categories would be sufficiently different, one being an order of magnitude smaller than the other, that in the synthesis, the priorities of the elements in the smaller set have little effect on the decision. We do not have space to show how some *undesirable* elements can be compared among themselves and gradually extended to compare them with *desirable* ones as above. Thus one can go from negatives to positives but keep the measurement of the two types positive, by eventually clustering them separately.

6. Synthesis: How to Combine Tangibles with Intangibles – Additive vs Multiplicative






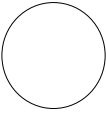
Let H be a complete hierarchy with h levels. Let B_k be the priority matrix of the k th level, $k = 2, \dots, h$. If W' is the global priority vector of the p th level with respect to some element z in the $(p-1)$ st level, then the priority vector W of the q th level ($p < q$) with respect to z is given by the multilinear (and thus nonlinear) form,

$$W = B_q B_{q-1} \dots B_{p+1} W'.$$

The global priority vector of the lowest level with respect to the goal is given by,

$$W = B_h B_{h-1} \dots B_2 W'.$$

In general, $W' = I$. The sensitivity of the bottom level alternatives with respect to changes in the weights of elements in any level can be studied by means of this multilinear form.

	.07		.28		.6 5
Unripe Tomato	Cherry	Small Green Tomato		Lime	
	.08		.22		.7 0
Lime	Grapefruit			Honeydew	
$\frac{.08}{.08} = 1$	$\frac{.08}{.08} = 2.75$			$\frac{.08}{.08} = 8.75$	
.65H1=.65	.65H2.75=1.79			.65H8.75=5.69	

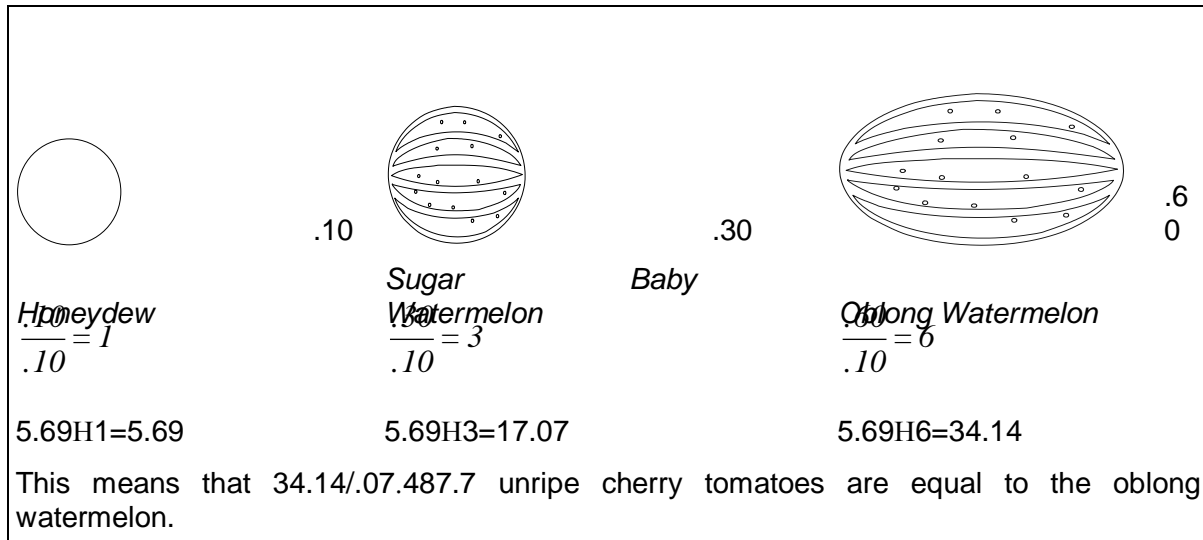


Figure 6. Comparisons According to Volume

Assume that a family is considering buying a house and there are three houses to consider A, B, and C. Four factors dominate their thinking: the price of the house, the remodeling costs, the size of the house as reflected by its footage and the style of the house which is an intangible. They have looked at three houses with numerical data shown below on the quantifiables (Figure 7):

Choosing the Best House				
	Price (\$1000)	Remodeling Costs (\$300)	Size (sq. ft.)	Style
A	200	150	3000	Colonial
B	300	50	2000	Ranch
C	500	100	5500	Split Level

Figure 7. Ranking Houses on Four Criteria

If we add the costs on price and modeling and normalize we obtain respectively (A,B,C) = (.269,.269,.462). Now let us see what is needed for normalization to yield the same result.

First we normalize for each of the quantifiable factors. Then we must normalize the factors measured with respect to a single scale (Figure 8).

Choosing the Best House				
	Price (1000/1300)	Remodeling Costs (300/1300)	Size (sq. ft.)	Style
A	200/1000	150/300	3000	Colonial
B	300/1000	50/300	2000	Ranch
C	500/1000	100/300	5500	Split Level

Figure 8. Normalization of the Measurements

Here we learn two important lessons to be used in the general approach. Normalizing the alternatives for the two criteria involving money in terms of the money involved on both criteria leads to relative weights of importance for the criteria. Here for example Price is in the ratio of about three to one when compared with Remodeling Cost and when compared with the latter with respect to the goal of choosing the best house, it is likely to be assigned the value “moderate” which is nearly three times more as indicated by the measurements. Here the criteria Price and Remodeling Cost derive their priorities only from the alternatives because they are equally important factors, although they can also acquire priorities from higher level criteria as to their functional importance with respect to the ease and availability of different amounts of money. We now combine the two factors with a common scale by weighting and adding. We have (Figure 9):

Choosing the Best House						
	Economic Factors (combining Price and Remodeling Cost)		Additive Synthesis	Multiplicative Synthesis	Size (sq. ft.)	Style
A	350/1300	=	.269	.256	3000/10500	Colonial
B	350/1300	=	.269	.272	2000/10500	Ranch
C	600/1300	=	.462	.472	5500/10500	Split Level

Figure 9. Combining the Two Costs through Additive or Multiplicative Syntheses

The left column and its decimal values in the second column give the exact value of the normalized dollars spent on each house obtained by additive synthesis (weighting and adding). By aggregating the two factor measured with dollars into a single factor, one then makes the decision as to which house to buy by comparing the three criteria as to their importance with respect to the goal.

The second lesson is that *when the criteria have different measurements, their importance cannot be determined from the bottom up through measurement of the alternatives, but from the top down, in terms of the goal. The same process of comparison of the criteria with respect to the goal is applied to all criteria if, despite the presence of a physical scale, they are assumed to be measurable on different scales as they might when actual values are unavailable or when it is thought that such measurement does not reflect the relative importance of the alternatives with respect to the given criterion. Imagine that no physical scale of any kind is known!* We might note in passing that the outcome of this process of comparison with respect to higher level criteria yields meaningful (not arbitrary) results as noted by two distinguished proponents of multi-attribute value theory (MAVT) Buede and Maxwell (1995), who wrote about their own experiments in decision making:

These experiments demonstrated that the MAVT and AHP techniques, when provided with the same decision outcome data, very often identify the same alternatives as 'best'. The other techniques are noticeably less consistent with MAVT, the Fuzzy algorithm being the least consistent.

Multiplicative synthesis, as in the third column of numbers above, done by raising each number in the two columns in the previous table to the power of its criterion measured in the relative total dollars under it, multiplying the two outcomes for each alternative and normalizing, *does not* yield the *exact answer* obtained by adding dollars! In addition, A and B should have the same value, but they do not with multiplicative synthesis. The multiplicative “solution” devised for the fallacy of always preserving rank and avoiding inconsistency fails, because it violates the most basic of several requirements mentioned in the introduction to this paper.

Multiplicative and additive syntheses are related analytically through approximation. If we denote by a_i the priority of the i th criterion, $i = 1, \dots, n$, and by x_i , the priority of alternative x with respect to the i th criterion, then

$$\begin{aligned} \prod x_i^{a_i} &= \exp \log \prod x_i^{a_i} = \exp(\sum \log x_i^{a_i}) = \exp(\sum a_i \log x_i) \approx 1 + \sum a_i \log x_i \\ &\approx 1 + \sum (a_i x_i - a_i) = \sum a_i x_i \end{aligned}$$

If desired, one can include a remainder term to estimate the error. With regard to additive and multiplicative syntheses being close, one may think that in the end it does not matter which one is used, but it does. Saaty and Hu (1998) have shown that despite such closeness on every matrix of consistent judgments in a decision, the synthesized outcomes by the two methods not only lead to different final priorities (which can cause a faulty allocation of resources) but more significantly to *different rankings* of the alternatives. For all these problems, but more significantly because it does not generalize to dependence and feedback even with consistency guaranteed, and because of the additive nature of matrix multiplication needed to compute feedback in network circuits to extend the AHP to the ANP, I do not recommend ever using multiplicative synthesis. It can lead to an undesirable ranking of the alternatives of a decision.

7. Rank Preservation and Reversal

Given the assumption that the alternatives of a decision are completely independent of one another, can and should the introduction (deletion) of new (old) alternatives change the rank of some alternatives without introducing new (deleting old) criteria, so that a less preferred alternative becomes most preferred? Incidentally, how one prioritizes the criteria and subcriteria is even more important than how one does the alternatives which are themselves composites of criteria. Can rank reverse among the criteria themselves if new criteria are introduced? Why should that not be as critical a concern? The answer is simple. In its original form utility theory assumed that criteria could not be weighted and the only important elements in a decision were the alternatives and their utilities under the various criteria. Today utility theorists imitate the AHP by rating, and some even by comparing the criteria, somehow. There was no concern then about what would happen to the ranks of the alternatives should the criteria weights themselves change as there were none. The tendency, even today, is to be unconcerned about the theory of rank preservation and reversal among the criteria themselves.

The house example of the previous section teaches us an important lesson. If we add a fourth house to the collection, the priority weights of the criteria Price and Remodeling Cost would change accordingly. Thus the measurements of the alternatives and their number which we call structural factors, always affect the importance of the criteria. When the criteria are incommensurate and their functional priorities are determined in terms of yet higher level criteria or goals, one must still weight such functional importance of the criteria by the structural effect of the alternatives. What is significant in all this is that the importance of the criteria always depends on the measurements of the alternatives. If we assume that the alternatives are measured on a different scale for each criterion, it becomes obvious that normalization is the instrument that provides the structural effect to update the importance of the criteria in terms of what alternatives there are. Finally, the priorities of the alternatives are weighted by the priorities of the criteria that depend on the measurements of the alternatives. This implies that the overall ranking of any alternative depends on the measurement and number of all the alternatives. To always preserve rank means that the priorities of the criteria should not depend on the measurements of the alternatives but should only derive from their own functional importance with respect to higher goals. This implies that

the alternatives should not depend on the measurements of other alternatives. Thus one way to always preserve rank is to rate the alternatives one at a time. In the AHP this is done through absolute measurement with respect to a complete set of intensity ranges with the largest value intensity value equal to one. It is also possible to preserve rank in relative measurement by using an ideal alternative with full value of one for each criterion.

The logic about what can or should happen to rank when the alternatives *depend* on each other has always been that *anything* can happen. Thus, when the criteria functionally depend on the alternatives, which implies that the alternatives, which of course depend on the criteria, would then depend on the alternatives themselves, rank may be allowed to reverse. The Analytic Network Process (ANP) is the generalization of the AHP to deal with ranking alternatives when there is functional dependence and feedback of any kind. Even here, one can have a decision problem with dependence among the criteria, but with no dependence of criteria on alternatives and rank may still need to be preserved. The ANP takes care of functional dependence, but if the criteria do not depend on the alternatives, the latter are kept out of the supermatrix and ranked precisely as they are dealt with in a hierarchy (Saaty, 1996).

Examples of rank reversal abound in practice, and they do not occur because new criteria are introduced. The requirement that rank always be preserved or that it should be preserved with respect to irrelevant alternatives. To every rule or generalization that one may wish to set down about rank, it is possible to find a counterexample that violates that rule. Here is the last and most extreme form of four variants of an attempt to qualify what should happen to rank given by Luce and Raiffa, each of which is followed by a counterexample. They state it but and then reject it. *The addition of new acts to a decision problem under uncertainty never changes old, originally non-optimal acts into optimal ones. The all-or-none feature of the last form may seem a bit too stringent ... a severe criticism is that it yields unreasonable results.* The AHP has a theory and implementation procedures and guidelines for when to preserve rank and when to allow it to reverse. One mode of the AHP allows an irrelevant alternative to cause reversal among the ranks of the original alternatives.

Guidelines for Selecting the Distributive or Ideal Mode

The distributive mode of the AHP produces preference scores by normalizing the performance scores; it takes the performance score received by each alternative and divides it by the sum of performance scores of all alternatives under that criterion. This means that with the Distributive mode the preference for any given alternative would go up if we reduce the performance score of another alternative or remove some alternatives.

The Ideal mode compares each performance score to a fixed benchmark such as the performance of the best alternative under that criterion. This means that with the Ideal mode the preference for any given alternative is independent of the performance of other alternatives, except for the alternative selected as a benchmark. Saaty and Vargas (1993) have shown by using simulation, that there are only minor differences produced by the two synthesis modes. This means that the decision should select one or the other if the results diverge beyond a given set of acceptable data.

The following guidelines were developed by Millet and Saaty (1999) to reflect the core differences in translating performance measures to preference measures of alternatives. ***The Distributive (dominance) synthesis mode should be used when the decision maker is concerned with the extent to which each alternative dominates all other alternatives under the criterion. The Ideal (performance) synthesis mode should be used when the decision maker is concerned with how well each alternative performs relative to a fixed benchmark.*** In order for dominance to be an issue the decision-maker should regard inferior alternatives as relevant even after the ranking process is completed. This suggests a simple test for the use of the Distributive mode: ***if the decision maker indicates that the preference for a top ranked alternative under a given criterion would improve if the performance of any lower ranked alternative***

was adjusted downward, then one should use the Distributive synthesis mode. To make this test more actionable we can ask the decision maker to imagine the amount of money he or she would be willing to pay for the top ranked alternative. If the decision maker would be willing to pay more for a top ranked alternative after learning that the performance of one of the lower-ranked alternatives was adjusted downward, then the Distributive mode should be used.

Consider selecting a car: Two different decision makers may approach the same problem from two different points of views even if the criteria and standards are the same. The one who is interested in "getting a well performing car" should use the Ideal mode. The one who is interested in "getting a car that stands out" among the alternatives purchased by co-workers or neighbors, should use the Distributive mode.

8. Group Decision Making

Here we consider two issues in group decision making. The first is how to aggregate individual judgments, and the second is how to construct a group choice from individual choices.

How to Aggregate Individual Judgments

Let the function $f(x_1, x_2, \dots, x_n)$ for synthesizing the judgments given by n judges, satisfy the

(i) Separability condition (S): $f(x_1, x_2, \dots, x_n) = g(x_1)g(x_2)\dots g(x_n)$

for all x_1, x_2, \dots, x_n in an interval P of positive numbers, where g is a function mapping P onto a proper interval J and is a continuous, associative and cancellative operation. [(S) means that the influences of the individual judgments can be separated as above.]

(ii) Unanimity condition (U): $f(x, x, \dots, x) = x$ for all x in P . [(U) means that if all individuals give the same judgment x , that judgment should also be the synthesized judgment.]

(iii) Homogeneity condition (H): $f(ux_1, ux_2, \dots, ux_n) = uf(x_1, x_2, \dots, x_n)$ where $u > 0$ and x_k, ux_k ($k=1,2,\dots,n$) are all in P . [For ratio judgments (H) means that if all individuals judge a ratio u times as large as another ratio, then the synthesized judgment should also be u times as large.]

(iv) Power conditions (P_p): $f(x_1^p, x_2^p, \dots, x_n^p) = f^p(x_1, x_2, \dots, x_n)$. [(P_2) for example means that if the k th individual judges the length of a side of a square to be x_k , the synthesized judgment on the area of that square will be given by the square of the synthesized judgment on the length of its side.]

Special case ($R=P_{-1}$): $f(1/x_1, 1/x_2, \dots, 1/x_n) = 1/f(x_1, x_2, \dots, x_n)$. [(R) is of particular importance in ratio judgments. It means that the synthesized value of the reciprocal of the individual judgments should be the reciprocal of the synthesized value of the original judgments.]

Aczel and Saaty (see Saaty 1990 and 1994) proved the following theorem:

Theorem *The general separable (S) synthesizing functions satisfying the unanimity (U) and homogeneity (H) conditions are the geometric mean and the root-mean-power. If moreover the reciprocal property (R) is assumed even for a single n -tuple (x_1, x_2, \dots, x_n) of the judgments of n individuals, where not all x_k are equal, then only the geometric mean satisfies all the above conditions.*

In any rational consensus, those who know more should, accordingly, influence the consensus more strongly than those who are less knowledgeable. Some people are clearly wiser and more sensible in such matters than others, others may be more powerful and their opinions should be given appropriately greater weight. For such unequal importance of voters not all g 's in (S) are the same function. In place of (S), the

weighted separability property (WS) is now: $f(x_1, x_2, \dots, x_n) = g_1(x_1)g_2(x_2) \dots g_n(x_n)$. [(WS) implies that not all judging individuals have the same weight when the judgments are synthesized and the different influences are reflected in the different functions (g_1, g_2, \dots, g_n).]

In this situation, Aczel and Alsina (see Saaty 1994) proved the following theorem:

Theorem *The general weighted-separable (WS) synthesizing functions with the unanimity (U) and homogeneity (H) properties are the weighted geometric mean $f(x_1, x_2, \dots, x_n) = x_1^{q_1} x_2^{q_2} \dots x_n^{q_n}$ and the weighted root-mean-powers $f(x_1, x_2, \dots, x_n) = \sqrt[\gamma]{q_1 x_1^\gamma + q_2 x_2^\gamma + \dots + q_n x_n^\gamma}$, where $q_1 + q_2 + \dots + q_n = 1$, $q_k > 0$ ($k=1, 2, \dots, n$), $\gamma > 0$, but otherwise $q_1, q_2, \dots, q_n, \gamma$ are arbitrary constants.*

If f also has the reciprocal property (R) and for a single set of entries (x_1, x_2, \dots, x_n) of judgments of n individuals, where not all x_k are equal, then *only the weighted geometric mean* applies. We give the following theorem which is an explicit statement of the synthesis problem that follows from the previous results, and applies to the second and third cases of the deterministic approach:

Theorem *If $x_1^{(i)}, \dots, x_n^{(i)}$ $i=1, \dots, m$ are rankings of n alternatives by m independent judges and if a_i is the importance of judge i developed from a hierarchy for evaluating the judges, and hence $\sum_{i=1}^m a_i = 1$, then $\left(\prod_{i=1}^{a_i} x_1^{a_i} \right), \dots, \left(\prod_{i=1}^{a_i} x_n^{a_i} \right)$ are the combined ranks of the alternatives for the m judges.*

The power or priority of judge i is simply a replication of the judgment of that judge (as if there are as many other judges as indicated by his/her power a_i), which implies multiplying his/her ratio by itself a_i times, and the result follows.

The first requires knowledge of the functions which the particular alternative performs and how well it compares with a standard or benchmark. The second requires comparison with the other alternatives to determine its importance.

On the Construction of Group Choice from Individual Choices

Given a group of individuals, a set of alternatives (with cardinality greater than 2), and individual ordinal preferences for the alternatives, Arrow proved with his Impossibility Theorem that it is impossible to derive a rational group choice (construct a social choice function that aggregates individual preferences) from ordinal preferences of the individuals that satisfy the following four conditions, i.e., at least one of them is violated:

Decisiveness: the aggregation procedure must generally produce a group order.

Unanimity: if all individuals prefer alternative A to alternative B, then the aggregation procedure must produce a group order indicating that the group prefers A to B.

Independence of irrelevant alternatives: given two sets of alternatives which both include A and B, if all individuals prefer A to B in both sets, then the aggregation procedure must produce a group order indicating that the group, given any of the two sets of alternatives, prefers A to B.

No dictator: no single individual preferences determine the group order.

Using the ratio scale approach of the AHP, it can be shown that because now the individual preferences are cardinal rather than ordinal, it is possible to derive a rational group choice satisfying the above four conditions. It is possible because:

- a) Individual priority scales can always be derived from a set of pairwise cardinal preference judgments as long as they form at least a minimal spanning tree in the completely connected graph formed by the elements being compared; and
- b) The cardinal preference judgments associated with group choice belong to a ratio scale that represents the relative intensity of preferences of the group.

REFERENCES

- Buede, D. and D.T. Maxwell, (1995), "Rank Disagreement: A Comparison of Multi-criteria Methodologies", *Journal of Multi-Criteria Decision Analysis*, Vol. 4, 1-21.
- Luce, R. D. and H. Raiffa, (1957), *Games and Decisions*, Wiley, New York.
- Millet, I. And T.L. Saaty, (1999), "On the Relativity of Relative Measures--Accommodating Both Rank Preservation and Rank Reversal in the AHP". *European Journal of Operational Research*.
- Peniwati, K., (1996), "The Analytic Hierarchy Process: The Possibility for Group Decision Making", pp. 202-214, *Proceedings of the Fourth International Symposium on the Analytic Hierarchy Process*, Vancouver, Canada. (Obtainable from RWS Publications, 4922 Ellsworth Avenue, Pittsburgh, PA 15213.)
- Saaty, T. L., (1999-2000 ed.), *Decision Making For Leaders*. RWS Publications, 4922 Ellsworth Avenue, Pittsburgh, PA 15213.
- Saaty, T. L., (2000), *The Brain, Unraveling the Mystery of How It Works: The Neural Network Process*, RWS Publications, 4922 Ellsworth Avenue, Pittsburgh, PA 15213.
- Saaty, T. L. and G. Hu, (1998), "Ranking by Eigenvector Versus Other Methods in the Analytic Hierarchy Process", *Appl. Math. Letters*, Vol. 11, No. 4, pp. 121-125.
- Saaty, T. L., (1996), *Decision Making with Dependence and Feedback: The Analytic Network Process*, RWS Publications, 4922 Ellsworth Avenue, Pittsburgh, PA 15213.
- Saaty, T. L., (1994), *Fundamentals of Decision Making and Priority Theory*, RWS Publications, 4922 Ellsworth Avenue, Pittsburgh, PA 15213.
- Saaty, T. L., (1990), *Multicriteria Decision Making: The Analytic Hierarchy Process*, RWS Publications, 4922 Ellsworth Avenue, Pittsburgh, PA.
- Saaty, T.L. and L. G. Vargas, (1993), "Experiments on Rank Preservation and Reversal in Relative Measurement," *Mathematical and Computer Modeling*, 17, No. 4/5, pp. 13-18.

Vargas, L. G., (1994), "Reply to Schenkerman's Avoiding Rank Reversal in AHP Decision Support Models", *European Journal of Operational Research*, 74, pp. 420-425.

Chapter 2

USING THE SUPERDECISIONS SOFTWARE TO BUILD AHP HIERARCHICAL DECISION MODELS

This chapter is a tutorial on how to create hierarchical decision models using the *SuperDecisions* software. It covers the commands and concepts used in the software and in this chapter we show how to build Analytic Hierarchy Process (AHP) hierarchical decision models.

INTRODUCTION TO BASIC CONCEPTS

This lesson describes how to build the simplest decision model that has a goal, criteria and alternatives, make judgments (paired comparisons), and compute the results to find the best alternative.

CLUSTERS AND ELEMENTS

A hierarchical decision model has a goal, criteria that are evaluated for their importance to the goal, and alternatives that are evaluated for how preferred they are with respect to each criterion. An abstract view of such a hierarchy is shown in Figure 11. The goal, the criteria and the alternatives are all elements in the decision problem, or nodes in the model. The lines connecting the goal to each criterion means that the criteria must be pairwise compared for their importance with respect to the goal. Similarly, the lines connecting each criterion to the alternatives mean the alternatives are pairwise compared as to which is more preferred for that criterion. Thus in the hierarchy that is shown there are six sets of pairwise comparisons, one for the criteria with respect to the goal and 5 for the alternatives with respect to the 5 criteria.

A *SuperDecisions* model consists of clusters of elements (or nodes), rather than elements (or nodes) arranged in levels. The simplest hierarchical model has a goal cluster containing the goal element, a criteria cluster containing the criteria elements and an alternatives cluster containing the alternative elements as shown in Figure 11. When clusters are connected by a line it means nodes in them are connected. The cluster containing the alternatives of the decision must be named *Alternatives*. Nodes and Clusters are organized alphabetically in the calculations, so an easy way to control the order is to preface the names with numbers.

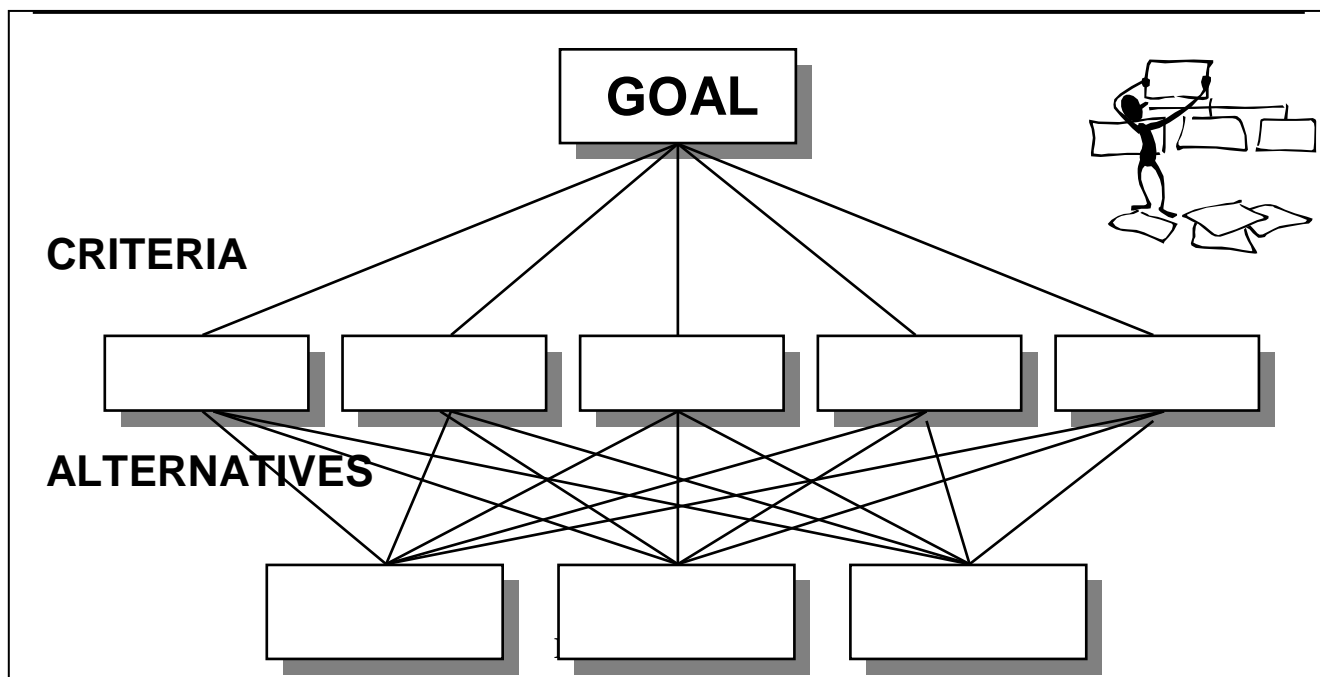


Figure 11. Abstract Representation of a Decision Hierarchy

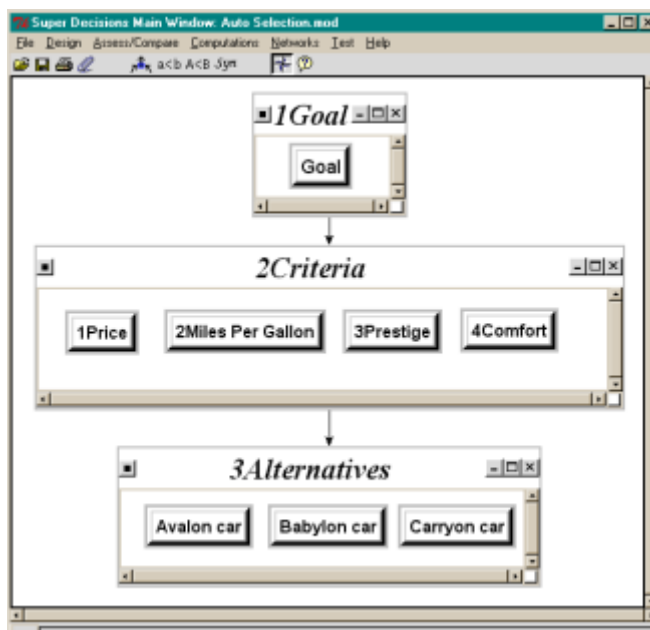



Figure 12. *SuperDecisions* Hierarchical Model for Selecting a Car.

The goal is connected to each of the criteria and each criterion is connected to each of the alternatives.

When clusters are shown connected with a line, it means elements in them are connected. To see which nodes are connected from a source or “parent” node, turn on the “Show Connections” icon  and place the cursor over the parent node. The nodes to which it is connected will then be outlined in red as shown in Figure 3.

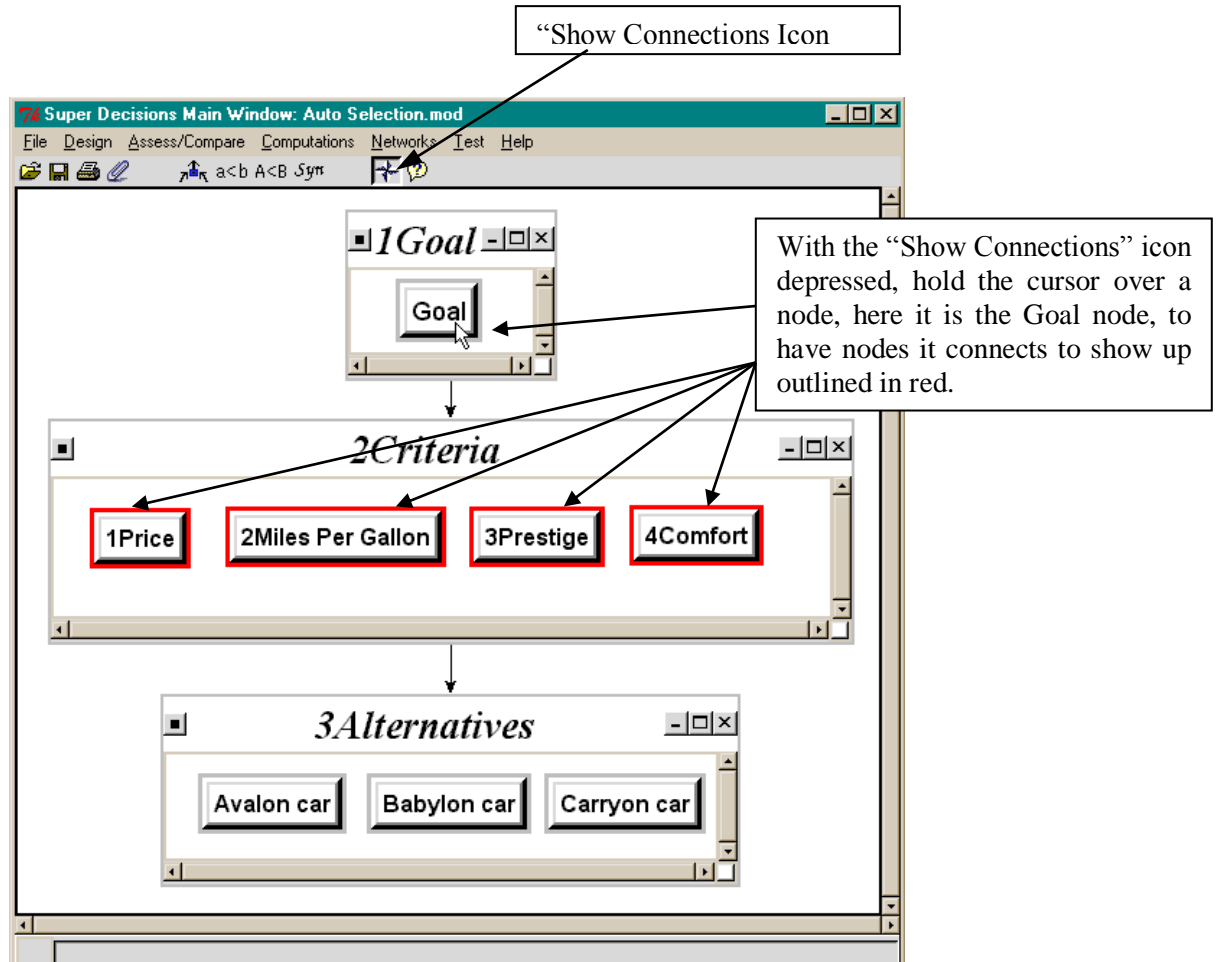


Figure 13. Turn on “Show Connections” Icon to see Element Connections.

Each criterion node is also connected to the alternatives. Place the cursor over each criterion in turn to highlight the alternatives for each.

In a hierarchical *SuperDecisions* model, the clusters are connected by arrows in one direction from top to bottom. In network models, covered in the next chapter, clusters may be connected with arrows going both ways and also may be connected to themselves with a loop.

In a hierarchical structure like that shown in Figure 13, each comparison set is made up of a parent node and the nodes it connects to in the cluster below. There are five sets of pairwise comparisons to do for this model: the criteria with respect to the goal, and the alternatives with respect to each of the four criteria.

Building a hierarchy is as much an art as it is a science. Following are some guidelines:

Guideline 1: Try not to include more than seven to nine elements in any cluster or grouping of elements because experiments have shown that it is cognitively challenging for human beings to deal with more than nine factors at one time and this can result in less accurate priorities.

Guideline 2: Try to cluster elements so that they include elements that are "comparable", or do not differ by orders of magnitude. In other words, try not to include items of very small significance in the same cluster as items of greater significance. The purpose of a hierarchy is to cluster the more important elements together and the less important elements together.

By keeping these two simple guidelines in mind, you will be able to model complex decisions correctly and efficiently.

DIFFERENTIATING BETWEEN OBJECTIVES AND CRITERIA

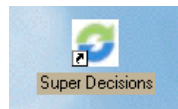
Often the words criteria and objectives are used interchangeably. A criterion is a principle or a standard that things are judged by while an objective is something that is sought or aimed for. The elements in a cluster may be thought of as objectives, or as criteria, depending on the model you are creating.

LESSON ONE - BUILDING A HIERARCHICAL MODEL

We will now show you how to build the model shown in **Error! Reference source not found.**

STARTING / CREATING A NEW MODEL (FILE)

Double click the *SuperDecisions* shortcut icon on the desktop that is shown below



or click the Windows Start key, select Programs, select *SuperDecisions* program group and then select *SuperDecisions*. A blank Main Window will appear

CREATE THE CLUSTERS

- From the Main Window menu, select Design, Cluster, New to create the first cluster. The new cluster dialogue box will appear as shown in Figure 14.
 - Shortcut: Press <Shift> <n> simultaneously, for the capital letter N), with your cursor is located on the background of the Main Window.
- Start the names of the clusters with numbers to control the order of the clusters as they are displayed in alphabetical order in the supermatrices.
- Type *1Goal* for the name and "This is the goal cluster, the top level in a hierarchical model" for the description. Select the "Create Another" button to create the next cluster. To change the font click on the buttons under Main Font and make your choices from the dropdown menus. For the model to appear as it does in **Error! Reference source not found.**, set the main font to New Times Roman, 28, Bold Italic.

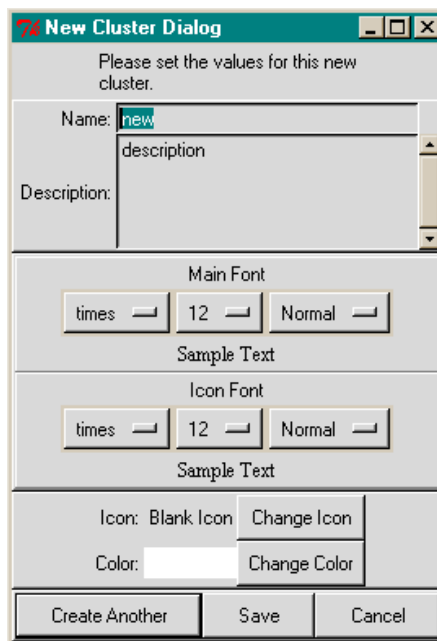
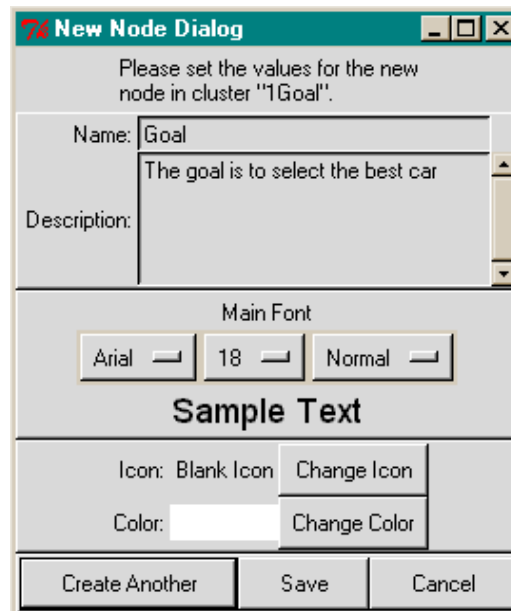


Figure 14. New Cluster Dialogue Box.

- Type *2Criteria* in the new cluster dialogue box with the description “Criteria for selecting a car,” and again select the “Create Another” button to create the next cluster.
- Type *3Alternatives* with the description “Alternative cars” and click the “Save” button to save and terminate the process of adding new clusters.

CREATE THE ELEMENTS INSIDE THE CLUSTERS

- Select Design, Node, New then select *1Goal* from the list of clusters that appears to show you wish to enter the element in that cluster. The New Node Dialogue box will appear.



- *Shortcut to create new node: Locate your cursor on the background of the goal cluster window and press the letter <n>.*
- *Another shortcut to create new node: Right click on the background of the cluster to get a drop down menu of commands and select “create node in cluster”*
- Type the word Goal and the description: “This is the goal element. The goal is to select the best car” as shown in Select the “Create Another” button to create the next cluster.
- To change the font click on the buttons under Main Font and select from the dropdown menus. For the model to appear as it does in **Error! Reference source not found.**, set the main font to Arial, 18, Normal.


ADD THE ELEMENTS AND THEIR DESCRIPTIONS TO THE OTHER CLUSTERS:

- **2Criteria Cluster**
 - 1Price: Purchase price of the cars
 - 2Miles per Gallon: miles per gallon
 - 3Prestige: Prestige car is perceived to bring to owner
 - 4Comfort: Comfort of the car
- **3Alternatives Cluster**
 - *Avalon Car:* Costs \$15,000, gets 26 mpg, low on prestige, good on comfort
 - *Babylon Car:* Costs \$18,000, gets 28 mpg, fair prestige, fair comfort
 - *Carryon Car:* Costs \$24,000, gets 20 MPG, high prestige, high comfort

CONNECT THE ELEMENTS

No lines connect the clusters yet because the elements within them are not connected.

TO CONNECT THE GOAL ELEMENT TO THE CRITERIA:

- Select the Design, “Node connections from” command. In the list that appears click on the node named Goal that is to be the parent or source node. In the list that now appears, left click on the criteria nodes to select them.
 - *Shortcut: To connect using the “Make Connections” icon , left click on the icon to depress it and turn on the connections mode, then left click on the source node and right click on each of the nodes it connects to. To turn off the connections mode, right click the icon again to un-depress it.*
 - *Shortcut: To disconnect, left click on the source node and right click on any of the nodes that are outlined in red (that were previously connected to it) to disconnect.*

TO CONNECT THE CRITERIA TO THE ALTERNATIVES:

- Select the Design, “Node connections from” command. In the list that appears click on the node named 1Price that is to be the parent or source node. In the list that now appears, left click on all the alternatives to select them.
- Connect the rest of the criterion nodes: 2Miles per Gallon, 3Prestige and 4Comfort, in turn.

SHORTCUT FOR CONNECTING ALL THE NODES IN A CLUSTER AT ONCE:

- Press the <Shift> key and hold it down while left clicking on one of the nodes in the cluster. This selects all the nodes in that cluster as source nodes. (*Make sure the “Make Connections” icon is turned on, that is, depressed*).
- Move to the cluster with nodes to be connected from the source nodes, press the <Shift> key and hold it down while right clicking on one of the nodes in the “to” cluster. This selects all the nodes and connects all the previously selected source nodes to them.

THE NODE MENU FOR RENAMING NODES, REMOVING THEM, MAKING AND HIGHLIGHTING CONNECTIONS, AND ENTERING THE COMPARISON MODE:

- Right click on a node to get its dropdown menu shown below from which you can perform operations on that node:

Edit node	e
Remove node	r
Node connexions from	F2
Hilight nodes connected to this one	F3
Hilight nodes connected from this one	F4
Unhighlight Nodes	Shift-F4
Node compare interface	F5
Make/show subnetwork	
Remove subnetwork	

SAVING YOUR MODEL

These commands for saving a model are located on the File menu.

- **File, Save** saves a copy of your model in the directory you designate as a file with the extension .sdmod
- **File, Save As** is used to save your model under a new name. The old file is closed and you continue working in the new file. It is useful to save successive versions such as car1.sdmod, car2.sdmod, car3.sdmod and so on about every 20 minutes so you always have a recent copy you can restore in case of an unforeseen problem.
- **File, Save As from a subnet** gives you the option of saving the whole model, or only the subnet (plus any sub-subnets branching down from it).
- **Files from previous versions of the software.** Models from previous versions may have been saved in a compacted format with the extension .mod.gz or the extension .sdmodzip. Such models can still be opened, not by double-clicking, but with the current version using the File, Open command.
- **Automatic backups** have been disabled and are no longer available. There is no un-do command. Please make frequent versioned backups, especially when constructing models or making judgments.
-

Next you will learn how to make judgments or pairwise comparisons about the criteria and alternatives in your model.

ASSESSMENTS / PAIRWISE COMPARISONS OVERVIEW

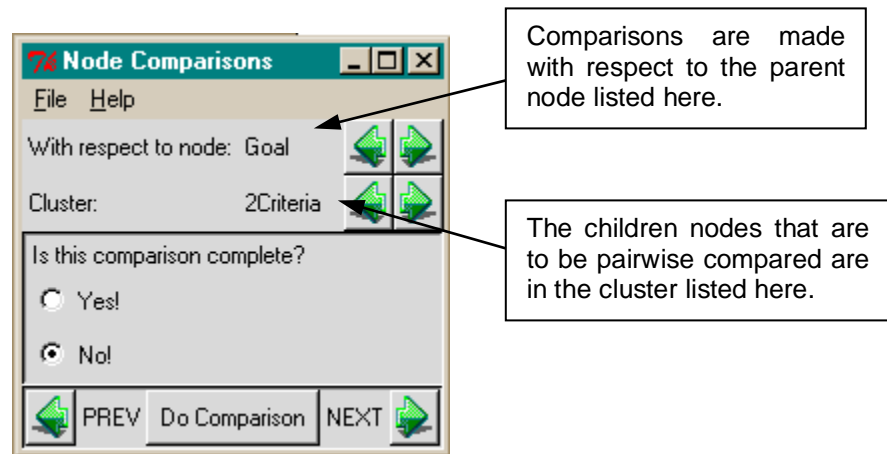
One of the major strengths of the AHP is the use of pairwise comparisons to derive accurate ratio scale priorities, as opposed to using traditional approaches of "assigning weights" which can also be difficult to

justify. Pairwise comparison is the process of comparing the relative importance, preference, or likelihood of two elements (for example, criteria) with respect to another element (for example, the goal) in the level above to establish priorities for the elements being compared. Pairwise comparisons are carried out for all the parent/children sets of nodes.

The nodes that are to be pairwise compared are always all in the same cluster and are compared with respect to their parent element, the node from which they are connected. This results in “local priorities” of the children nodes with respect to the parent.

Select the Assess/Compare, Node comparisons command to start the comparison process. If a node has been selected by clicking on it prior to starting the comparisons, that node will be the parent node in the first set of comparisons to come up. If no node has been selected, the first node in the first cluster will be the parent. The parent node and the cluster where the children nodes are located are specified in the node comparison dialogue box that appears.

Click on the Do Comparison button to start the comparison process. The pairwise comparison process starts in the mode that was last in use, or in the Questionnaire mode the first time. There are four pairwise comparison assessment modes. To switch from one mode to the other click on the tab at the top. When a judgment entered in one mode is recorded in one mode, the same judgment is recorded in all the modes. Calculations are based on the numbers in the Matrix mode (the Questionnaire mode always shows whole numbers, and no numbers are shown in the Graphic and Verbal mode)



- **Questionnaire** In this mode, select the judgment on the side of the most dominant member of the pair, that is, the most important, if it is a criterion as shown below, or the most preferred, in the case of alternatives.

Questionnaire Mode

Comparisons for "2Criteria" wrt "Goal"

File Computations Misc. Help

Graphic Verbal Matrix **Questionnaire**

1Price is moderately to strongly more important than 2Miles Per Gallon

1. 1Price	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	2Miles Per ~
2. 1Price	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	3Prestige
3. 1Price	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	4Comfort
4. 2Miles Per ~	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	3Prestige
5. 2Miles Per ~	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	4Comfort
6. 3Prestige	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	4Comfort

- **Numerical judgments** are made in a matrix using a nine-point scale that represent how many times one element is more important than another. The arrow at the left of the entry points to the dominant element. To switch which element is dominant, left double-click on the arrow.

Numerical or Matrix Mode

Comparisons for "2Criteria" wrt "Goal"

File Computations Misc. Help

Graphic Verbal **Matrix** Questionnaire

1Price is 4.0 times more important than 2Miles Per Gallon

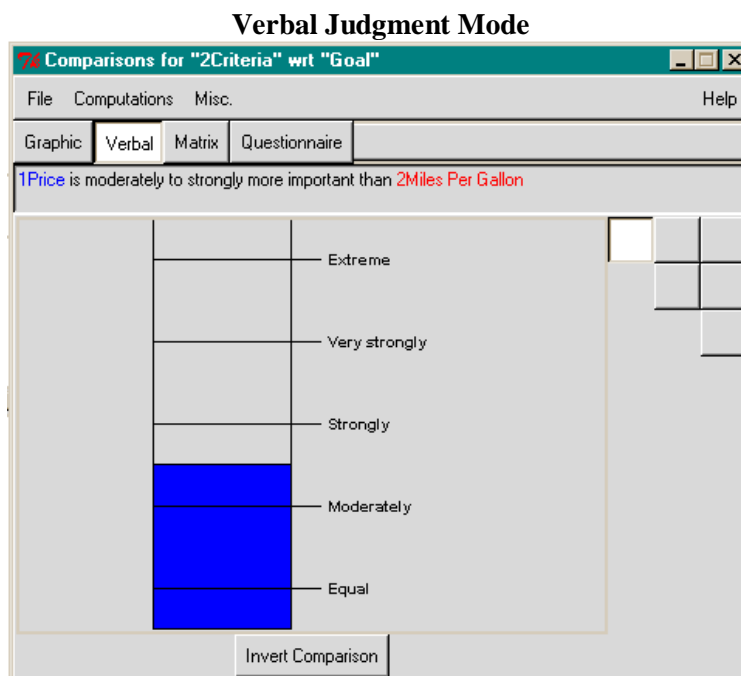
Inconsistency 2Miles Per ~ 3Prestige 4Comfort

1Price	← 4.0	← 6.0	← 4.0
2Miles Per ~		← 3.0	↑ 2.0
3Prestige			↑ 4.0

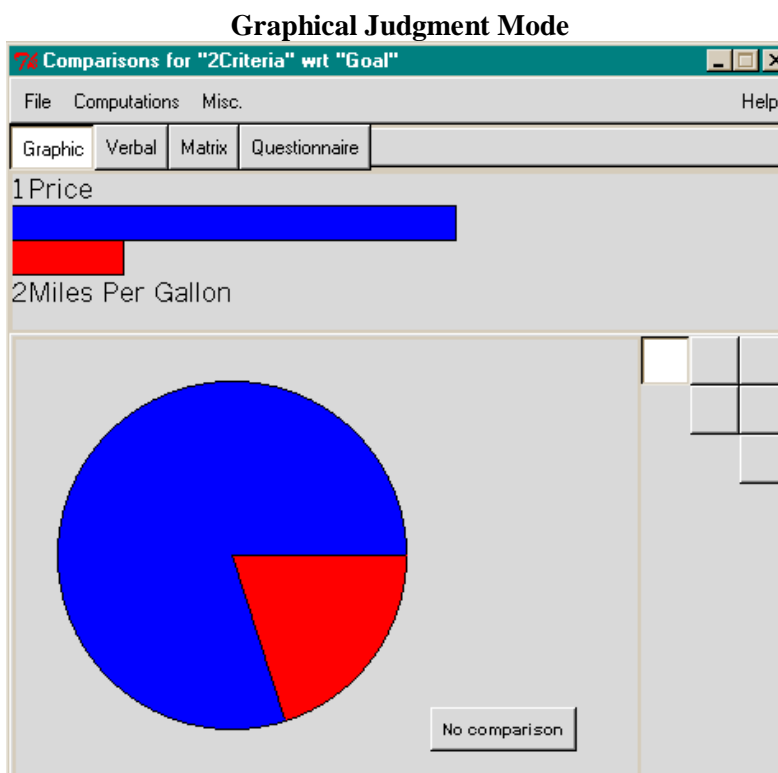
The current judgment is highlighted. Arrow points to dominant element.

Double click arrow to change dominant criterion from price to comfort.

- **Verbal judgments** are used to compare factors using the words Equal, Moderate, Strong, Very Strong, Extreme. Equal requires no explanation. Extreme means an order of magnitude – about 9 or 10 to 1. Judgments between these words, such as Moderate to Strong are also possible. In the verbal mode the node mentioned first is the dominant one. To invert click the “Invert Comparison” button at the bottom of the screen.



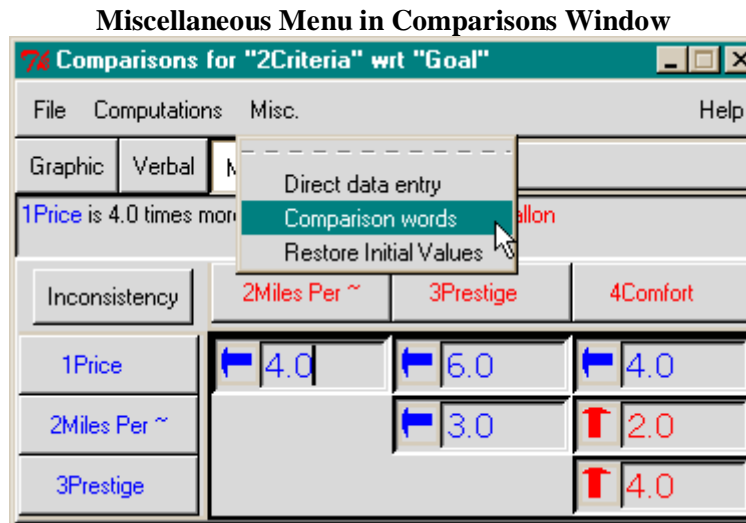
- **Graphical judgments** are made by clicking and dragging the pie slice to change the relative size of the pie slice to the circle and the relative length of the two bars until it represents how many times more important one element is than the other.



CHANGING THE PAIRED COMPARISON TYPE

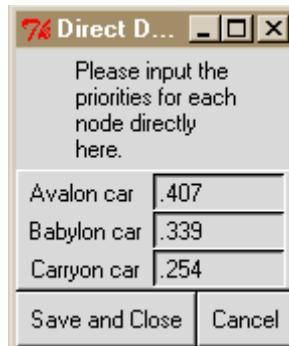
There are three different paired comparison types, **Importance**, **Preference** and **Likelihood**. Or you can select **Other** and enter a word of your own choosing – “beauty” for example. Importance is most appropriate when comparing criteria or criteria. Preference is appropriate when comparing alternatives with respect to a criterion. Likelihood is appropriate when comparing the likelihood of uncertain events or scenarios, such as in risk analysis.

- Select the command Misc./ Comparison words to change the comparison type.



- Select the command **Direct data entry** to directly enter data values that are converted to priorities by summing and dividing each entry by the total (normalizing to 1).

Entering Direct Priorities for Prices of Cars



When entering numbers for preference and higher numbers are better, enter the data itself. If higher numbers are less preferred, for example, for car prices, higher numbers (prices) are less desirable. In this case invert the priorities as shown below. A spreadsheet program, Excel, was used to perform the calculations and create the table below (E-05 means 10^{-5}). The final priorities show Avalon with the highest priority and Carryon the lowest, but their proportion is the same as it was originally.

Car	Price	1/Price	(1/Price)/Total (final priorities)
Avalon car	407	0.002457	0.407
Babylon car	339	0.002950	0.339
Carryon car	254	0.003937	0.254

Avalon	15000	6.667E-05	0.407
Babylon	18000	5.556E-05	0.339
Carryon	24000	4.167E-05	0.254
	Total	0.000164	

Tip: Even when you have data, as with car prices it is often better to interpret what the numbers mean to you and use the Questionnaire mode so the final results match your personal values or your personal interpretation of what the numbers mean to you.

Select the command Computations/Priorities to see the results from pairwise comparisons or from direct data such as prices of cars. Prices need to be “inverted” as shown above.

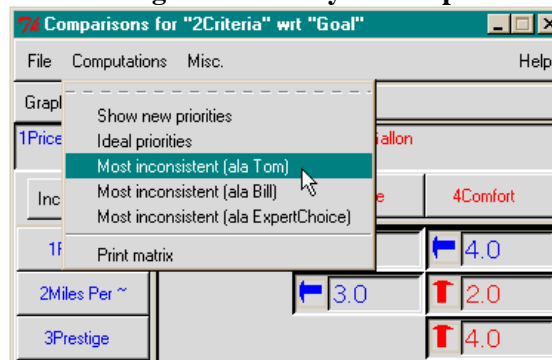
CHECKING INCONSISTENCY

The inconsistency measure is useful for identifying possible errors in judgments as well as actual inconsistencies in the judgments themselves; this is accessed from the Priorities Window.

Inconsistency measures the logical inconsistency of your judgments. For example, if you were to say that A is more important than B and B is more important than C and then say that C is more important than A you are not being consistent. A somewhat less inconsistent situation would arise if you would say that A is 3 times more important than B, B is 2 times more important than C, and that C is 8 times more important than A.

In general, the inconsistency ratio should be less than 0.1 or so to be considered reasonably consistent. To find out which judgment is most inconsistent select the Computations/Most inconsistent (ala Tom) to have it highlighted. Regardless of the mode you are in when you select that command, you will be taken to the Matrix mode. You will then be asked if you want to see the value that would make that judgment most consistent with the rest.

Menu for Checking Inconsistency in Comparison Window



You may enter the suggested value, or some other judgment that you think fits better, by typing it in, or leave it as it is.

JUDGING THE CRITERIA WITH RESPECT TO THE GOAL

Since judgments about the relative importance of the criteria may depend on the alternatives being considered, we recommend that judgments be made from the "bottom up", that is first for the alternatives, then for the sub-criteria, and then for the criteria. However, for illustrative purposes, we will make judgments "top down" in this tutorial.

- From the Main Screen, left click the Goal (until depressed).
- Then from the menu, select Assess/Compare, Node comparisons; then click the Do comparison button.

The Questionnaire pairwise window will appear. Enter the judgments as shown by clicking on the numbered cells. For example, clicking the 4 on the side of *Price* indicates that it is between moderately and strongly more important than *Miles per gallon*. After a judgment has been entered in each row select the Computations/Priorities command to get the priorities.

Judgments for Criteria with Respect to the Goal

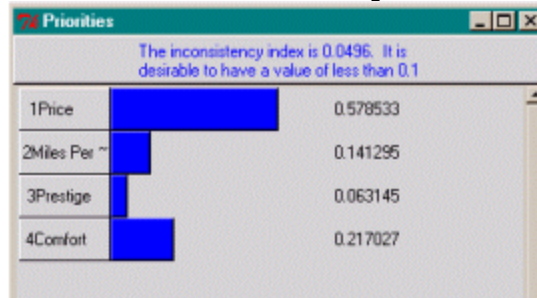
The screenshot shows a window titled "Comparisons for '2Criteria' wrt 'Goal'". It has a menu bar with "File", "Computations", "Misc.", and "Help". Below the menu bar are tabs for "Graphic", "Verbal", "Matrix", and "Questionnaire". The "Questionnaire" tab is selected. The main area displays a pairwise comparison matrix for four criteria: 1Price, 2Miles Per ~, 3Prestige, and 4Comfort. The matrix is a 4x4 grid of numbered cells (1-9) representing judgments. The diagonal cells are empty. The cells are numbered as follows:

	1Price	2Miles Per ~	3Prestige	4Comfort
1. 1Price		4	3	2
2. 2Miles Per ~	9		3	2
3. 3Prestige	9	8		2
4. 4Comfort	9	8	7	

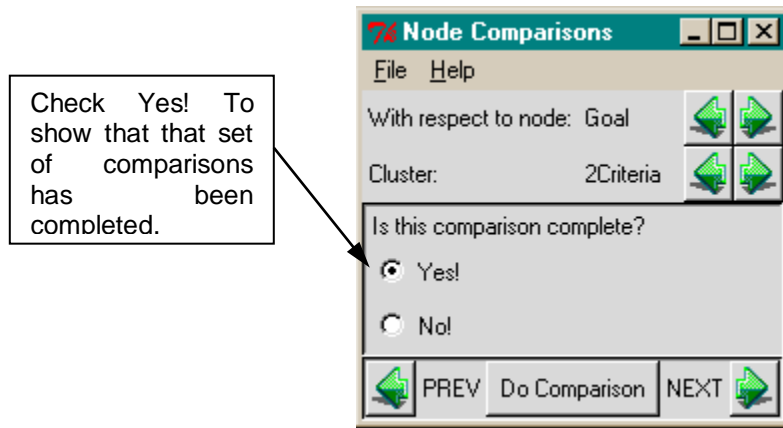
Below the matrix, there are four rows of text, each followed by a "No comp." button and a "4Comfort" button:

- 1Price is moderately to strongly more important than 2Miles Per ~
- 1Price is moderately to strongly more important than 3Prestige
- 1Price is moderately to strongly more important than 4Comfort
- 2Miles Per ~ is moderately to strongly more important than 3Prestige
- 2Miles Per ~ is moderately to strongly more important than 4Comfort
- 3Prestige is moderately to strongly more important than 4Comfort


Priorities for Criteria with Respect to the Goal



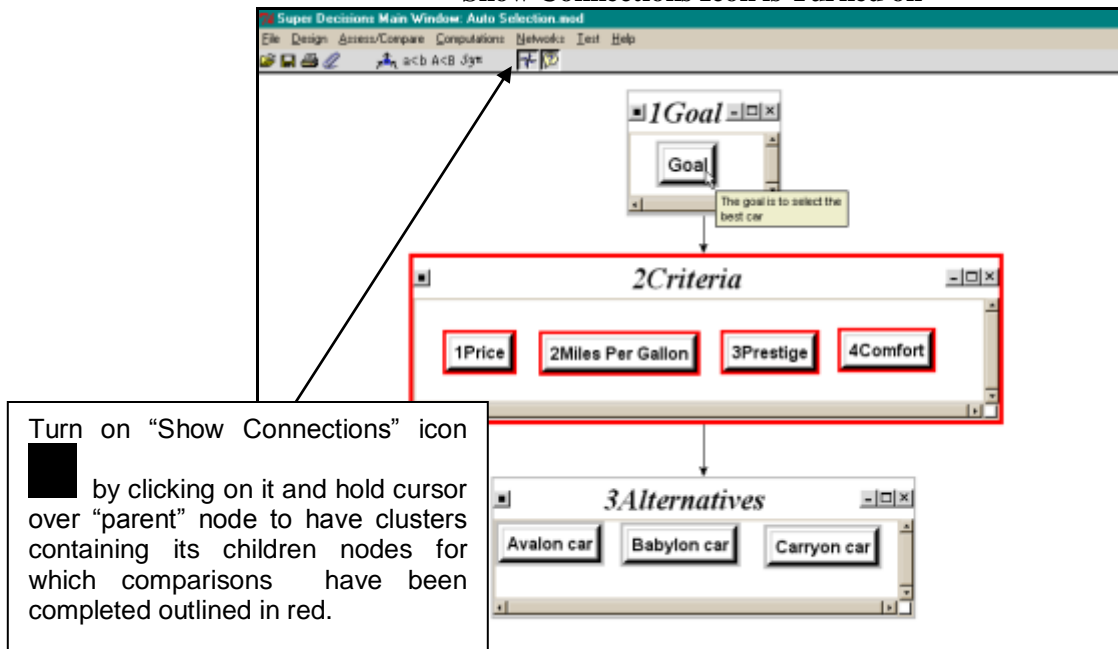
Click the X at the top right of the window, or select the File, Close command, to close the comparison window and return to the main screen. After the window closes, you will be given an opportunity to check the Yes box to note that that set of comparisons has been completed:



CHECKING COMPLETED COMPARISONS

To show which comparisons have been completed left click the “Show Node Comparisons” icon  and hold the cursor over the “parent node”, in this case the Goal, to have clusters with completed comparisons outlined in red.


Show Connections Icon is Turned on



Now you will make judgments about the cars with respect to the criteria. There will be four sets of comparisons, one for each criterion.

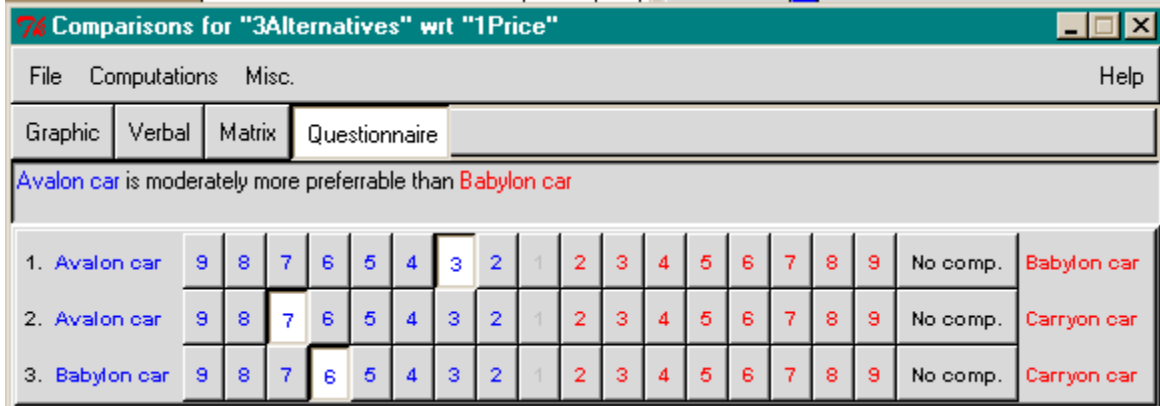
MAKING JUDGMENTS ABOUT THE ALTERNATIVES WITH RESPECT TO THE CRITERIA

Now make your judgments comparing the cars with respect to *Price*.

- Click on the *Price* node and the shortcut icon for node comparisons  and change the comparison word from *Importance* to *Preference* with the command Misc/Comparison words. The car prices are:
 - Avalon \$15,000
 - Babylon \$18,000
 - Carryon \$24,000

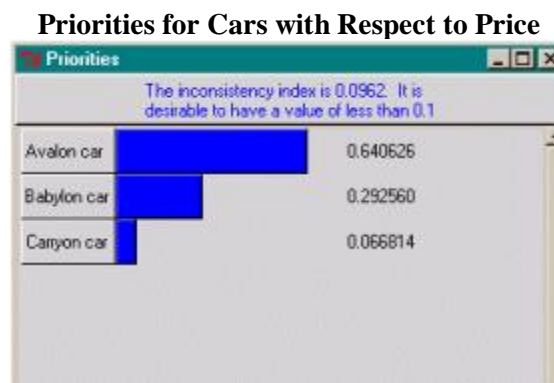
Since we are looking at cars in this inexpensive to moderate price range, we are sensitive to price. From our price-sensitive perspective, \$15K is moderately preferred to \$18K and very strongly preferred to \$24K, while \$18K is between strongly and very strongly preferred to \$24K. Enter the judgments shown below.

Judgments (in Terms of Preference) for Cars with Respect to Price



	1. Avalon car	2. Avalon car	3. Babylon car
1. Avalon car	9	8	7
2. Avalon car	8	7	6
3. Babylon car	7	6	5

To show the priorities resulting from these judgments select the Computations/Priorities command.



The inconsistency is shown as 0.0962 at the top of the screen. Inconsistencies that are less than 0.1 are acceptable. If it were greater than 0.1, then you should select the Computations/Inconsistency (a la Tom) command and try to improve it.

Comparing the priorities obtained with respect to price using judgments, versus the priorities obtained earlier by using direct price data we have:

Cars	Priorities obtained using judgments to interpret preference for prices	Priorities obtained by “inverting” the actual prices and normalizing.
Avalon	.641	0.407
Babylon	.293	0.339
Carryon	.067	0.254

You can see from the results above that the preference for the Avalon with respect to *Price* is much greater based on the judge’s personal interpretation of what the car prices mean in that individual’s financial situation.

2. Now enter the judgments for Miles per Gallon. The data are:
 - Avalon 26 mpg
 - Babylon 28 mpg
 - Carryon 20 mpg

Judgments Showing our Preferences for Cars with Respect to Miles per Gallon

Comparisons for "3 Alternatives" wrt "2 Miles Per Gallon"

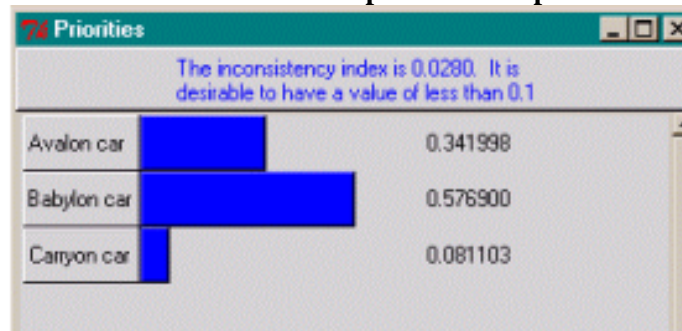
File Computations Misc. Help

Graphic Verbal Matrix Questionnaire

Babylon car is strongly to very strongly more preferable than Carryon car

1. Avalon car	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	Babylon car
2. Avalon car	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	Carryon car
3. Babylon car	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	Carryon car

Priorities with Respect to Miles per Gallon



3. Now enter the judgments for *Prestige*. The data are:
 - Avalon low
 - Babylon fair
 - Carryon high

Judgments (in Terms of Preference) for Cars with Respect to Prestige

Comparisons for "3Alternatives" wrt "3Prestige"

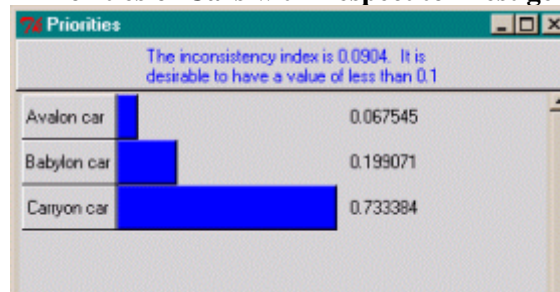
File Computations Misc. Help

Graphic Verbal Matrix **Questionnaire**

Carryon car is strongly more preferable than Babylon car

1. Avalon car	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	Babylon car
2. Avalon car	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	Carryon car
3. Babylon car	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	Carryon car

Priorities of Cars with Respect to Prestige



4. Finally, make the judgments for the cars with respect to *Comfort*. The data are:
- Avalon Good comfort
 - Babylon Fair comfort
 - Carryon Hi comfort

Judgments (in Terms of Preference) for Cars with Respect to Comfort

Comparisons for "3Alternatives" wrt "4Comfort"

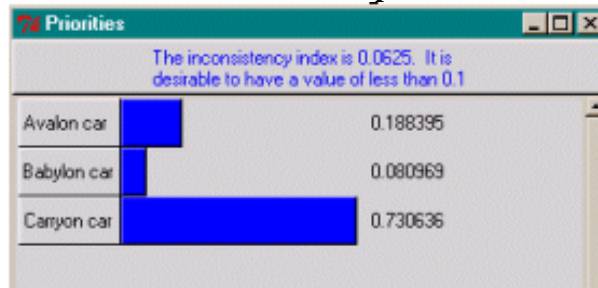
File Computations Misc. Help

Graphic Verbal Matrix **Questionnaire**

Carryon car is very strongly more preferable than Babylon car

1. Avalon car	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	Babylon car
2. Avalon car	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	Carryon car
3. Babylon car	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	Carryon car

Priorities of Cars with Respect to Comfort



All the prioritizing is now finished. There is a command for comparing clusters, but in a hierarchic model the clusters are not compared.

AFTER JUDGMENTS ARE MADE AND PRIORITIES ARE DERIVED

The priorities are entered in the Unweighted Supermatrix. Select the Computations/Unweighted SuperMatrix/ Graphical to see how the priorities that were derived in the comparison process appear there.

Unweighted Supermatrix


Cluster Node Labels		1Goal	2Criteria				3Alternatives		
		Goal	1Price	2Miles Per Gallon	3Prestige	4Comfort	Avalon car	Babylon car	Carryon car
1Goal	Goal	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
2Criteria	1Price	0.578536	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	2Miles Per Gallon	0.141294	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	3Prestige	0.063142	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	4Comfort	0.217027	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
3Alternatives	Avalon car	0.000000	0.640637	0.341998	0.188394	0.188394	0.000000	0.000000	0.000000
	Babylon car	0.000000	0.292561	0.576905	0.080961	0.080961	0.000000	0.000000	0.000000
	Carryon car	0.000000	0.066802	0.081097	0.730645	0.730645	0.000000	0.000000	0.000000
Done									

In a hierarchical model such as this one, the Weighted Supermatrix is the same as the Unweighted Supermatrix because the clusters are not weighted. Raising the Weighted Supermatrix to powers yields the Limit Matrix from which the final answers are extracted. The final priorities for the Alternatives are in the column under the Goal.

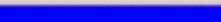


The Limit Matrix is the Final Supermatrix

Super Decisions Main Window: Auto Selection.mod: Limit Matrix									
Cluster Node Labels		1Goal	2Criteria				3Alternatives		
		Goal	1Price	2Miles Per Gallon	3Prestige	4Comfort	Avalon car	Babylon car	Carryon car
1Goal	Goal	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
2Criteria	1Price	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	2Miles Per Gallon	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	3Prestige	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	4Comfort	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
3Alternatives	Avalon car	0.471736	0.640637	0.341998	0.188394	0.188394	1.000000	0.000000	0.000000
	Babylon car	0.273453	0.292561	0.576905	0.080961	0.080961	0.000000	1.000000	0.000000
	Carryon car	0.254810	0.066802	0.081097	0.730645	0.730645	0.000000	0.000000	1.000000
Done									

SYNTHESIS - GETTING THE RESULTS

The results for the alternatives are obtained with the Synthesis command in the Main Model View. Select the Computations/Synthesize command, or click the shortcut icon  to see the final results:

Results from the Synthesis Command

New synthesis for: Super Decisions Main Window: Auto S...					
Here are the overall synthesized priorities for the alternatives. You synthesized from the network Super Decisions Main Window: Auto Selection.mod					
Name	Graphic	Ideals	Normals	Raw	
Avalon car		1.000000	0.471736	0.471736	
Babylon car		0.579674	0.273454	0.273454	
Carryon car		0.540154	0.254810	0.254810	

The *Normals* column presents the results in the form of priorities. This is the usual way to report on results. The *Ideals* column is obtained from the *Normals* column by dividing each of its entries by the largest value in the column. The *Raw* column is read directly from the Limit Supermatrix. For a hierarchical model such as this one, the *Raw* column and the *Normals* column are the same.

These results show that the Avalon would be the best car for this decision maker. This result also matches our intuition as we know a high priority was put on the price of the car.

RATINGS MODELS OVERVIEW

Applications such as personnel evaluation, resource allocation and college admission decisions, may involve a large number of alternatives. Making pairwise comparisons in a model that has a large number of alternatives can be very time consuming, as you would have to perform a large number of comparisons for the alternatives (up to $n*(n-1)/2$) with respect to each covering criterion. Instead of making pairwise comparisons for the alternatives, the data grid can be used.

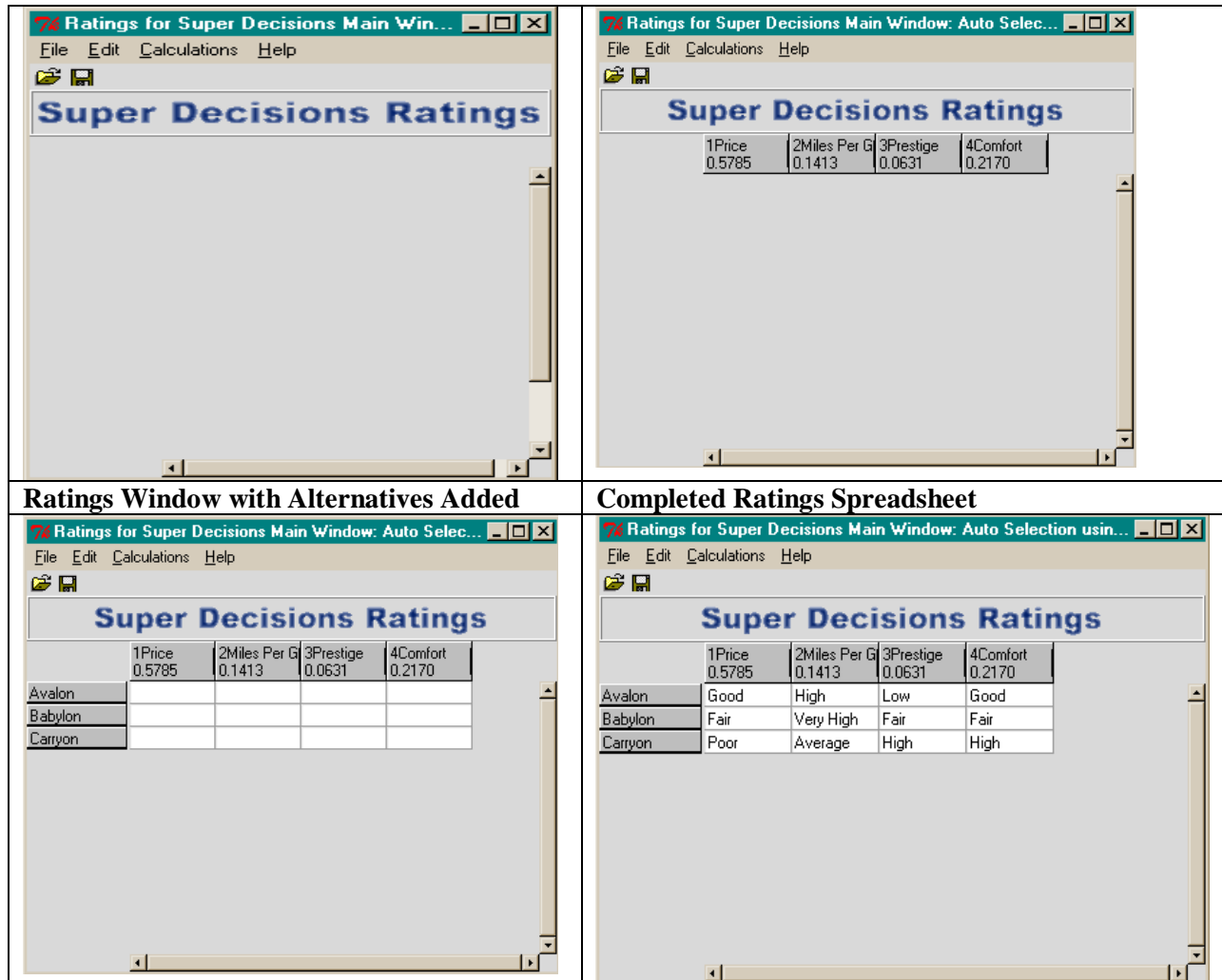
The use of the data grid combines the power of the hierarchy and the pairwise comparison process with the ability to evaluate hundreds or even thousands of alternatives. Pairwise comparisons are still used to evaluate the elements in the hierarchy itself, but not for evaluating the alternatives. Alternatives' priorities are established relative to each covering using rating categories.

WHEN TO USE RATINGS

People often "rate" alternatives using words such as High, Medium and Low; or Excellent, Very Good, Good, Fair and Poor with respect to some characteristic they have. We use the same idea and rate alternatives with respect to the criteria. We add one twist and prioritize the ratings words themselves, so that a "High" rating, for example, is twice as good as a "Medium" rating.

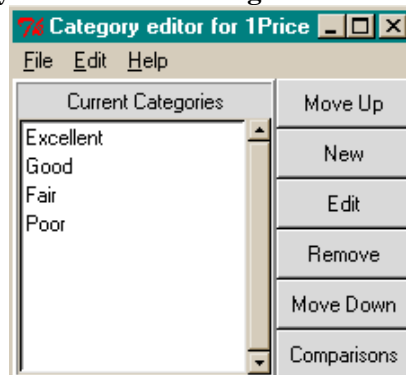
- **Create Ratings Spreadsheet** As an example, we shall convert the car selection model to a Ratings model. First, save a copy of the model by selecting the File/Save As command and name it "Auto Selection using Ratings". Remove the Alternatives cluster by right clicking on the title bar and deleting it. They will be put back into the Ratings spreadsheet. Select the Assess/Compare Ratings command to create the Ratings spreadsheet. Once the Ratings screen appears, shown below, the Main Model View will disappear and remain "locked out" until the Ratings screen is closed.
- **Add Criteria** Select the Edit Criteria New command and click on the nodes from the main model that will be the criteria in the Ratings spreadsheet. Selecting all four criteria brings them into the Ratings spreadsheet. Their priorities, shown below the names, were established in the main model though pairwise comparison.
- **Add Alternatives** Next, select the Edit Alternatives New command and type the names of the alternatives Avalon, Babylon and Carryon. This results in the Super Decisions Ratings table with alternatives added.

Startup Ratings Window	Ratings Window after Criteria Selected
------------------------	--



- Creating and Prioritizing the Ratings Categories** This step should be done before rating the alternatives. Select the Edit Criteria Categories command, choose *Price* and create categories for it (below):

Category Editor for Creating Criterion Categories



Click this button to pairwise compare categories and establish priorities for them.

Clicking the Comparison button will bring up the following comparison screen where we pairwise compare to establish priorities for the categories. Select the command Computations/Ideal Priorities to see the resulting priorities.

Compare the Rating Categories for Price for Ratings

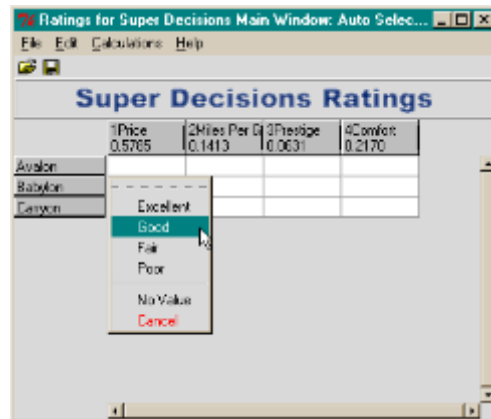
	9	8	7	6	5	4	3	2	1		
1. Excellent	9	8	7	6	5	4	3	2	1	2	3
2. Excellent	9	8	7	6	5	4	3	2	1	2	3
3. Excellent	9	8	7	6	5	4	3	2	1	2	3
4. Good	9	8	7	6	5	4	3	2	1	2	3
5. Good	9	8	7	6	5	4	3	2	1	2	3
6. Fair	9	8	7	6	5	4	3	2	1	2	3

Priorities for Ratings Categories	Idealized Priorities for Ratings Categories																
<p>The inconsistency index is 0.0984. It is desirable to have a value of less than 0.1</p> <table> <tr> <td>Excellent</td> <td>0.634959</td> </tr> <tr> <td>Good</td> <td>0.209698</td> </tr> <tr> <td>Fair</td> <td>0.109753</td> </tr> <tr> <td>Poor</td> <td>0.045589</td> </tr> </table>	Excellent	0.634959	Good	0.209698	Fair	0.109753	Poor	0.045589	<p>The inconsistency index is 0.0984. It is desirable to have a value of less than 0.1</p> <table> <tr> <td>Excellent</td> <td>1.000000</td> </tr> <tr> <td>Good</td> <td>0.330254</td> </tr> <tr> <td>Fair</td> <td>0.172851</td> </tr> <tr> <td>Poor</td> <td>0.071798</td> </tr> </table>	Excellent	1.000000	Good	0.330254	Fair	0.172851	Poor	0.071798
Excellent	0.634959																
Good	0.209698																
Fair	0.109753																
Poor	0.045589																
Excellent	1.000000																
Good	0.330254																
Fair	0.172851																
Poor	0.071798																

The idealized priorities shown above are obtained from the priorities in the first column above by dividing each by the largest (0.634959 in this case). The idealized priorities in the second column are used in the Ratings spreadsheet. For example, a rating of Good actually means use the number .330254 in the cell, multiply times the column priority and add across for all cells in the row to get the total for that row.

Close all open windows to return to the Ratings spreadsheet. Right or left click on the (Avalon, Price) cell to get the dropdown menu for selecting the appropriate category, in this case, *Good*.

Category Menu for Selecting Ratings in the Price Column



Create categories for the rest of the criteria and prioritize them. The rest of the categories that were established in this model and their priorities are shown in the table below. It is not necessary to have the same number of rating categories for each column (there are 3 for Prestige).

Price	Miles per Gallon	Prestige	Comfort
Excellent (.635)	Very High (.608)	High (.674)	High (.590)
Good (.210)	High (.267)	Fair (.226)	Good (.255)
Fair (.110)	Average (.087)	Low (.101)	Fair (.115)
Poor (.046)	Low (.037)		Low (.040)

VIEWING THE RESULTS IN RATINGS

- Select the Calculations/Totals command to see the evaluation scores before normalizing. The Total for each row is obtained by multiplying the values for the ratings in the cells times the column priorities and summing.
- Select the Calculations/Priorities command to see the results as priorities that sum to 1. The results are quite close in this example, but that is a coincidence.

RESULTS SHOWN AS TOTALS IN THE RATINGS SPREADSHEET	TOTALS CONVERTED TO PRIORITIES IN THE RATINGS SPREADSHEET

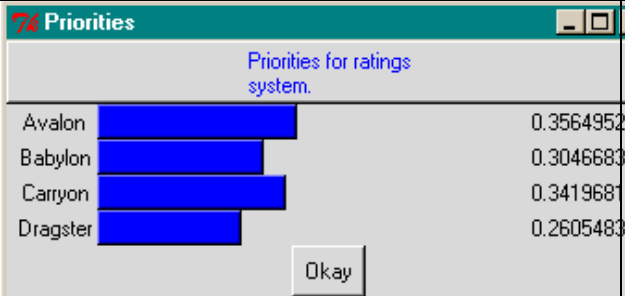
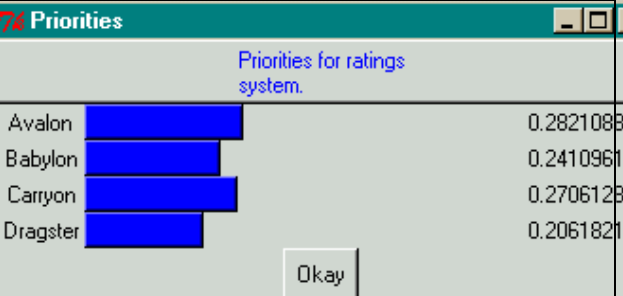
- Select the Calculations/Matrix Priorities command to see the ideal priority values in each cell that correspond to the verbal ratings.

74 Ratings Priorities Matrix				
	0.578536 1Price	0.141294 2Miles Per ~	0.063142 3Prestige	0.217027 4Comfort
Avalon	0.330254	0.439466	0.149380	0.432687
Babylon	0.172845	1.000000	0.334716	0.194639
Carryon	0.071799	0.143391	1.000000	1.000000

To see it is a coincidence that the totals and priorities are so close in this example, let us add another car, a Dragster, to the Ratings spreadsheet and rate it, then examine the totals and priorities. As you see below, the Totals and Priorities for the four cars are quite different.

Add Another Car and Rate It

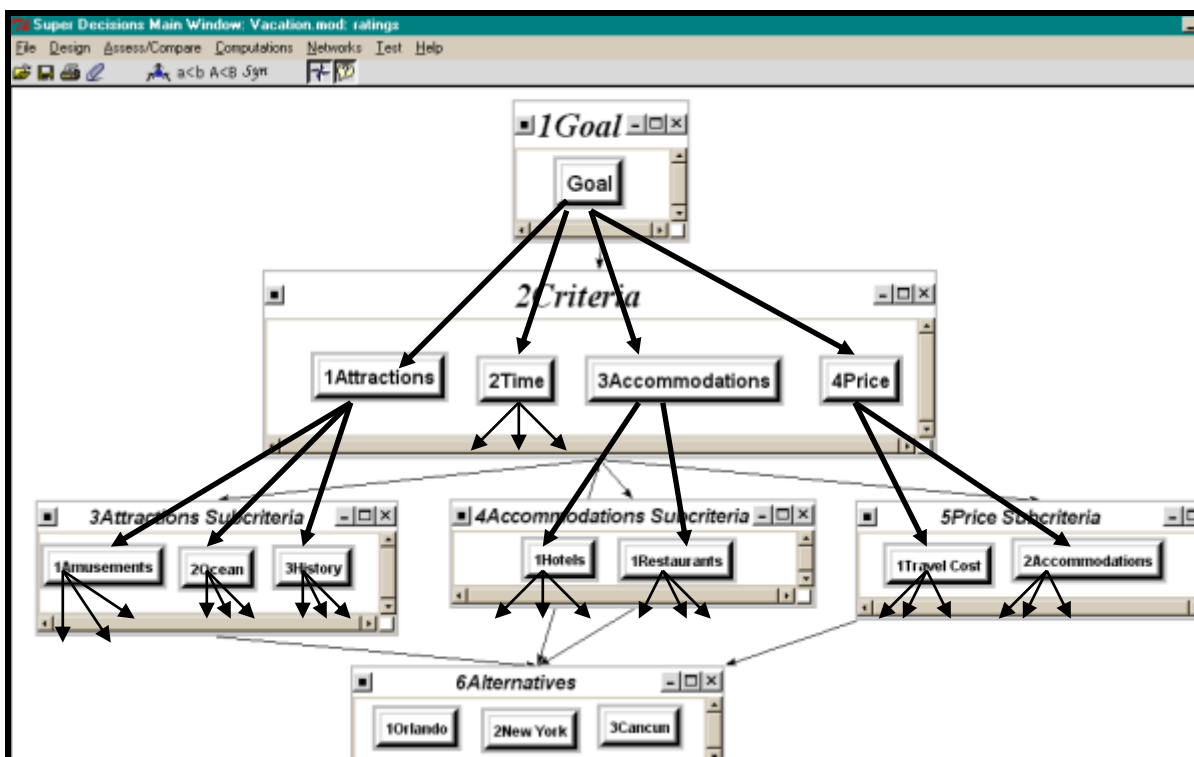
74 Ratings for Super Decisions Main Window: Auto Sel...				
File Edit Calculations Help				
Super Decisions Ratings				
	1Price 0.5785	2Miles Per G 0.1413	3Prestige 0.0631	4Comfort 0.2170
Avalon	Good	High	Low	Good
Babylon	Fair	Very High	Fair	Fair
Carryon	Poor	Average	High	High
Dragster	Poor	Very High	High	Low

Total Scores with Four Cars		Totals Converted to Priorities	
			
Avalon	0.35649526	Avalon	0.282108806
Babylon	0.30466834	Babylon	0.241096114
Carryon	0.34196810	Carryon	0.270612892
Dragster	0.26054831	Dragster	0.206182189

For a hierarchical model the results obtained in the Ratings Window are transferred to the Main Screen for the Synthesis command and the synthesized results are the same as the Ratings Priorities.

HIERARCHICAL MODELS WITH SUB-CRITERIA

When the criteria in a hierarchical model are broken down into sub-criteria one way to handle it, though not the only one is to put the criteria in a cluster, then create a cluster for the subcriteria of each criterion. A second way would be to put all the subcriteria in a level into the same cluster, but then it would be difficult to see at a glance the subcriteria that belong to a criterion.

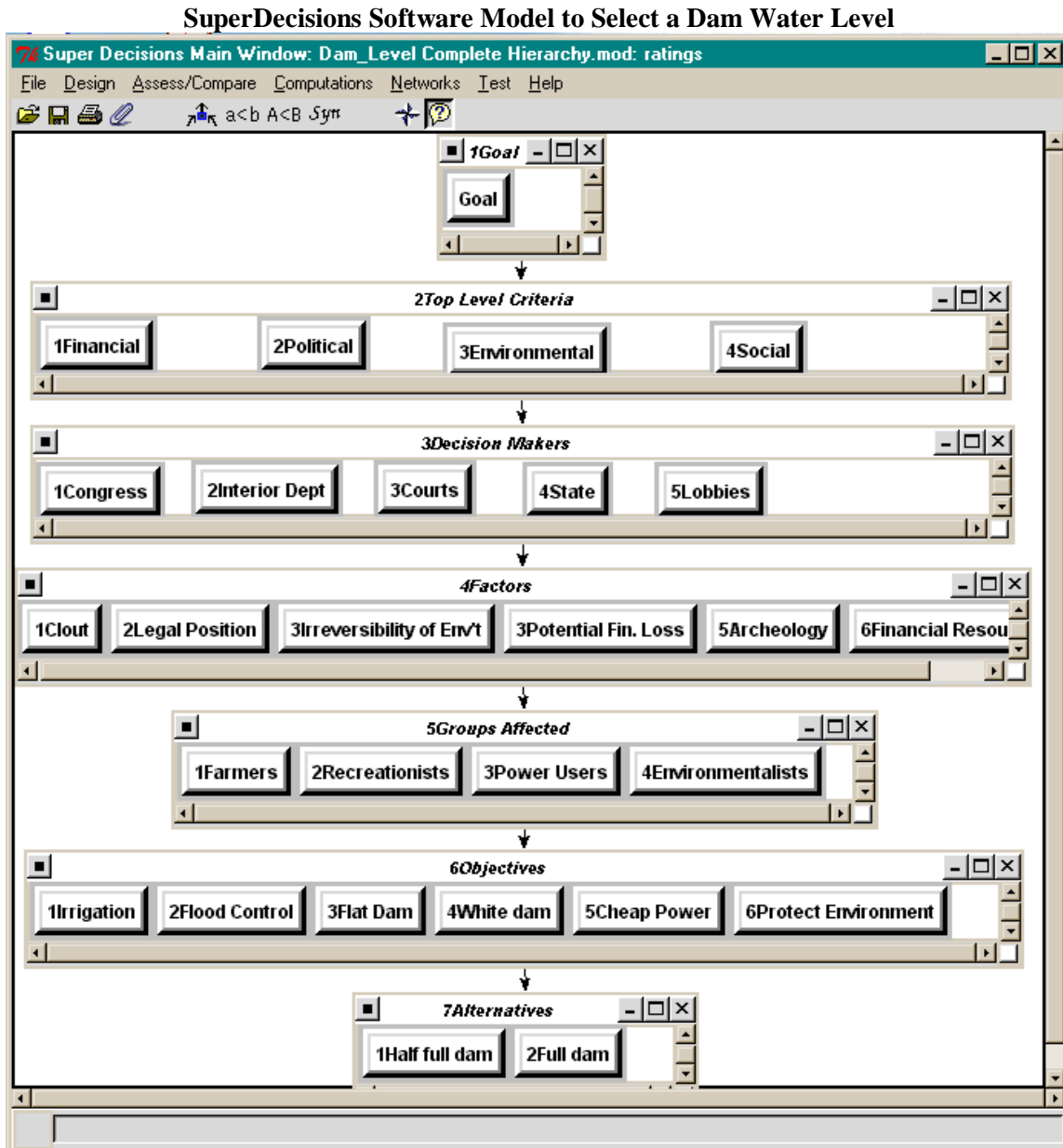


In the figure above, one main criterion node, Time, connects directly to the alternatives. The other criteria connect to subcriteria that then connect to the alternatives. The lowest level of criteria that connect to the alternatives are called the “covering criteria”. Here the covering criteria are the main criterion Time plus the subcriteria: Amusements, Oceans, History, Hotels, Restaurants, Travel Cost,

Accommodations. The pairwise comparisons are done in the usual way for this model. It can also be done as a Ratings model by selecting the bottom level or “covering criteria” as the criteria in Ratings.

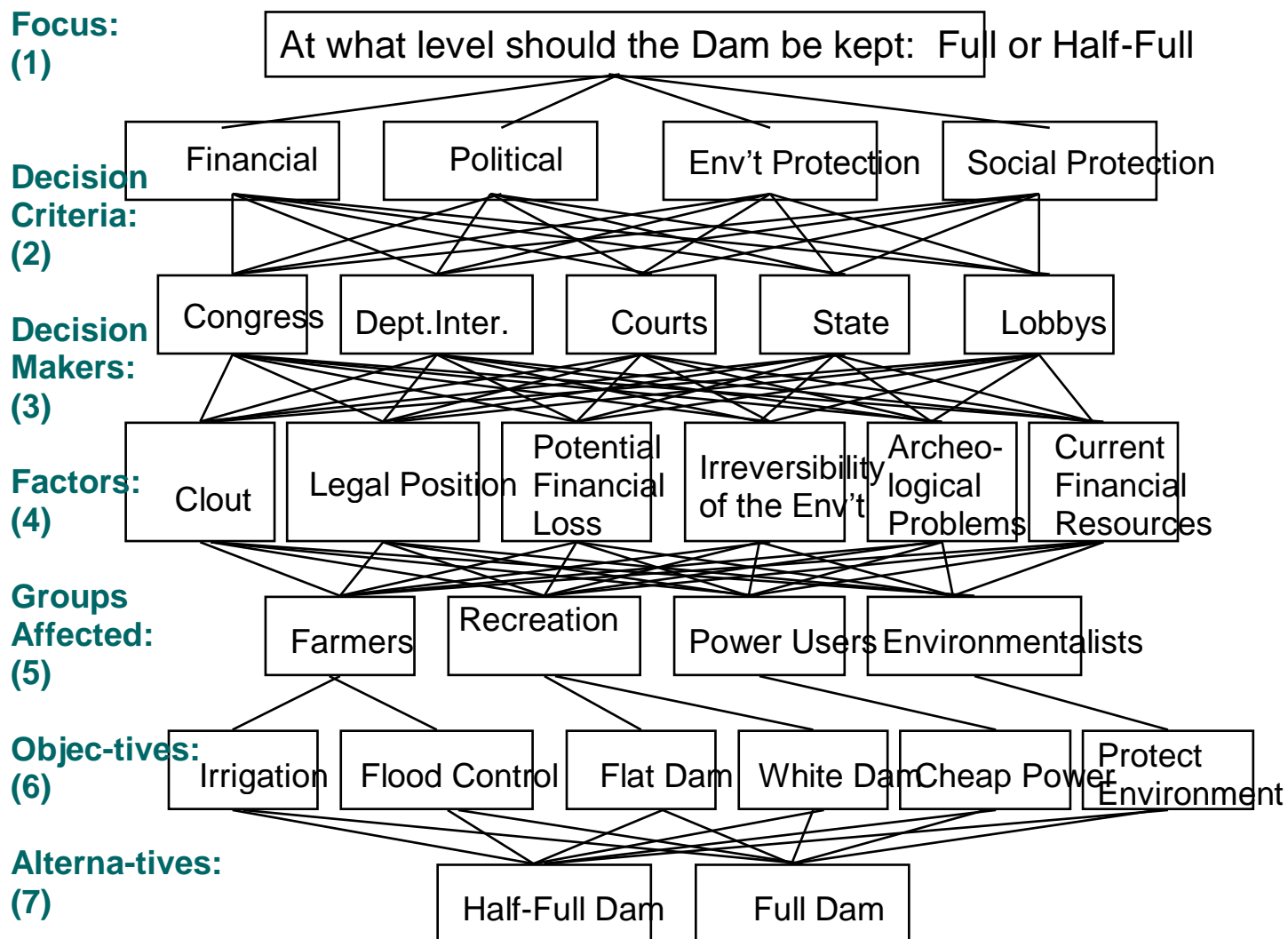
COMPLETE HIERARCHICAL MODELS

In a complete hierarchical model, the goal connects to the criteria, and the criteria connect to all the sub-criteria and all the sub-criteria connect to all the sub-sub criteria and so on. In the ANP software, a connection from one cluster to another means at least one node in the source cluster is connected to one node in the target cluster. To see exactly which nodes are connected in the software one must use the “Show Connections” feature. The following is an example of an almost-complete model as it appears in the ANP software. The node connections are shown for this model in the schematic in the next figure. The clusters in the ANP model correspond to the levels in the schematic that follows.



It is a complete hierarchy except between the “Groups Affected” and “Objectives” levels. There the Farmers were interested only in Irrigation and Flood Control, the Recreationists in Flat Dam (dam full of water) and White Dam (low water level in dam), Power Users were interested only in power and the Environmentalists only in the Environment. There are, however, complete connections from each of the other levels to the level below it.

Schematic of the Previous Model for Deciding on Water Level in a Dam



This model is almost a complete hierarchy except that the “Groups Affected” nodes in level 5 do not all connect to all the “Objectives” nodes in level 6. But you can see that it is quite easy to build a complete hierarchy type of model with the *SuperDecisions* software.

Chapter 3

USING THE SUPER DECISIONS SOFTWARE TO BUILD ANP MODELS

Summary

An Analytic Network Model of a problem may consist of a single network or a number of networks. To build an Analytic Network Process (ANP) network, you need to:

1. Think about the elements in it and decide what kind of logical groupings of nodes and clusters would best describe the problem;
2. Build a cluster first, then create the nodes within it
3. Select one node as a potential parent node and examine all the clusters in turn to determine if they have nodes that the parent node either influences or is influenced by to select its children nodes in that cluster.
4. Create the links between the parent node and all its children nodes in each cluster – this is how the comparison sets of nodes are created.
5. Clusters are linked automatically when nodes are linked.
6. Make sure the *influences* or *is influenced by* question is posed in a consistent way throughout this network. Make pairwise comparison judgments on nodes and clusters and synthesize.

The following topics are covered in the software tutorial:

- Installing the *ANP Software*
- A demonstration of a single network system (*Hamburger* model)
- A demonstration of a two-layer system (*Car Purchase BCR* model with Benefits Costs and Risks subnets) and of a three-layer system (*National Missile Defense* model) with its decision subnets in the bottom layer.
- Step-by-step walkthrough of building a simple single-network model
- Step-by-step walkthrough of building a complex model
- Special *SuperDecisions* software functions and commands

INTRODUCTION TO DECISION MAKING USING THE ANALYTIC NETWORK PROCESS (ANP)

The power of the Analytic Network Process (ANP) lies in its use of ratio scales to capture all kinds of interactions and make accurate predictions, and, even further, to make better decisions. So far, it has proven itself to be a success when expert knowledge was used with it to predict sports outcomes, economic turns, business, social and political events.

The ANP is the first mathematical theory that makes it possible for us to deal systematically with all kinds of dependence and feedback. The reason for its success is the way it elicits judgments and uses measurement to derive ratio scales. Priorities as ratio scales are a fundamental kind of number amenable to performing the basic arithmetic operations of adding within the same scale and multiplying different scales meaningfully as required by the ANP.

The Analytic Network Process (ANP) is a new theory that extends the AHP to cases of dependence and feedback and generalizes on the supermatrix approach introduced in Thomas Saaty's 1980 book on the Analytic Hierarchy Process. It allows interactions and feedback within clusters (inner dependence) and between clusters (outer dependence). Feedback can better capture the complex effects of interplay in human society. The ANP provides a thorough framework to include clusters of elements connected in any desired way to investigate the process of deriving ratio scales priorities from the distribution of influence among elements and among clusters. The AHP becomes a special case of the ANP. Although many decision problems are best studied through the ANP, it is not true that forcing an ANP model always yields better results than using the hierarchies of the AHP. There are examples to justify the use of both. We have yet to learn when the shortcut of the hierarchy is justified, not simply on grounds of expediency and efficiency, but also for reasons of validity of the outcome.

The ANP is implemented in the software *SuperDecisions* and has been applied to various problems both to deal with decisions and to illustrate the uses of the new theory. The ANP is a coupling of two parts. The first consists of a control hierarchy or network of criteria and subcriteria that control the interactions in the system under study. The second is a network of influences among the elements and clusters. The network varies from criterion to criterion and a supermatrix of limiting influence is computed for each control criterion. Finally, each of these supermatrices is weighted by the priority of its control criterion and the results are synthesized through addition for all the control criteria.

In addition, a problem is often studied through a control hierarchy or system of benefits, a second for costs, a third for opportunities, and a fourth for risks. The synthesized results of the four control systems are combined by taking the quotient of the benefits times the opportunities to the costs times the risks to determine the best outcome. Other formulas may be employed at times to combine results. Here is a rough outline of the steps of the ANP followed by two lists of concepts of both the AHP and the ANP.

CONCEPTS OF THE ANALYTIC HIERARCHY PROCESS

1. Elements of the problem, goal, subgoals, time horizons, scenarios, actors and stakeholders, their objectives and policies, criteria, subcriteria, attributes, and alternatives.
2. Hierarchic structure.
3. Judgments - absolute numbers, homogeneity, clustering, pivot elements, tangibles and intangibles.
4. Comparisons, dominance and reciprocity with respect to an attribute, inconsistency and the eigenvector, use of actual measurements.
5. The number of judgments; how to take fewer judgments.

6. Derived ratio scales - in AHP the priorities are derived and are proven to belong to a ratio scale.
7. Interval judgments, stochastic judgments.
8. Synthesis - multilinear forms - density.
9. Rank - the dominance mode, the performance mode with respect to an ideal.
10. Absolute measurement - rating alternatives one at a time.
11. Benefits, opportunities, costs and risks hierarchies.
12. Parallel with human thinking - neural firing creates awareness and intensity of stimuli for both tangibles and intangibles. Measurements are data to be interpreted.
13. Group Decision Making and the reciprocal property; Pareto optimality: if each prefers A to B, then the group does.
14. Sensitivity Analysis.
15. Learning and revision as a process.

CONCEPTS OF THE ANALYTIC NETWORK PROCESS

1. Feedback, inner and outer dependence.
2. Influence with respect to a criterion.
3. The control hierarchy or system.
4. The supermatrix.
5. The limiting supermatrix and limiting priorities.
6. Primitivity, irreducibility, cyclicity.
7. Make the limiting supermatrix stochastic: why clusters must be compared.
8. Synthesis for the criteria of a control hierarchy or a control system.
9. Synthesis for benefits, costs, opportunities, and risks control hierarchies.
10. Formulation to compute the limit.
11. Relation to Neural Network Firing - the continuous case.
12. The density of neural firing and distributions and their applications to reproduce visual images and symphonic compositions. Further research in the area is needed.

OUTLINE OF STEPS TO SOLVE A PROBLEM USING THE ANP

1. Describe the decision problem in detail including its objectives, criteria and subcriteria, actors and their objectives and the possible outcomes of that decision. Give details of influences that determine how that decision may come out.
2. Determine the control criteria and subcriteria in the four control hierarchies one each for the benefits, opportunities, costs and risks of that decision and obtain their priorities from paired comparisons matrices. If a control criterion or subcriterion has a global priority of 3% or less, you may consider carefully eliminating it from further consideration. The software automatically deals only with those criteria or subcriteria that have subnets under them. For benefits and opportunities, ask what gives the most benefits or presents the greatest opportunity to influence fulfillment of that control criterion. For costs and risks, ask what incurs the most cost or faces the greatest risk. Sometimes (very rarely), the comparisons are made simply in terms of benefits, opportunities, costs, and risks in the aggregate without using control criteria and subcriteria.
3. Determine the most general network of clusters (or components) and their elements that applies to all the control criteria. To better organize the development of the model as well as you can, number and arrange the clusters and their elements in a convenient way (perhaps in a column).

Use the identical label to represent the same cluster and the same elements for all the control criteria.

4. For each control criterion or subcriterion, determine the clusters of the general feedback system with their elements and connect them according to their outer and inner dependence influences. An arrow is drawn from a cluster to any cluster whose elements influence it.
5. Determine the approach you want to follow in the analysis of each cluster or element, influencing (the preferred approach) other clusters and elements with respect to a criterion, or being influenced by other clusters and elements. The sense (being influenced or influencing) must apply to all the criteria for the four control hierarchies for the entire decision.
6. For each control criterion, construct the supermatrix by laying out the clusters in the order they are numbered and all the elements in each cluster both vertically on the left and horizontally at the top. Enter in the appropriate position the priorities derived from the paired comparisons as subcolumns of the corresponding column of the supermatrix.
7. Perform paired comparisons on the elements within the clusters themselves according to their influence on each element in another cluster they are connected to (outer dependence) or on elements in their own cluster (inner dependence). In making comparisons, you must always have a criterion in mind. Comparisons of elements according to which element influences a given element more and how strongly more than another element it is compared with are made with a control criterion or subcriterion of the control hierarchy in mind.
8. Perform paired comparisons on the clusters as they influence each cluster to which they are connected with respect to the given control criterion. The derived weights are used to weight the elements of the corresponding column blocks of the supermatrix. Assign a zero when there is no influence. Thus obtain the weighted column stochastic supermatrix.
9. Compute the limit priorities of the stochastic supermatrix according to whether it is irreducible (primitive or imprimitive [cyclic]) or it is reducible with one being a simple or a multiple root and whether the system is cyclic or not. Two kinds of outcomes are possible. In the first all the columns of the matrix are identical and each gives the relative priorities of the elements from which the priorities of the elements in each cluster are normalized to one. In the second the limit cycles in blocks and the different limits are summed and averaged and again normalized to one for each cluster. Although the priority vectors are entered in the supermatrix in normalized form, the limit priorities are put in idealized form because the control criteria do not depend on the alternatives.
10. Synthesize the limiting priorities by weighting each idealized limit vector by the weight of its control criterion and adding the resulting vectors for each of the four merits: Benefits (B), Opportunities (O), Costs (C) and Risks (R). There are now four vectors, one for each of the four merits. An answer involving marginal values of the merits is obtained by forming the ratio BO/CR for each alternative from the four vectors. The alternative with the largest ratio is chosen for some decisions. Companies and individuals with limited resources often prefer this type of synthesis.
11. Governments prefer this type of outcome. Determine strategic criteria and their priorities to rate the four merits one at a time. Normalize the four ratings thus obtained and use them to calculate the overall synthesis of the four vectors. For each alternative, subtract the costs and risks from the

sum of the benefits and opportunities. At other times one may add the weighted reciprocals of the costs and risks. Still at other times one may subtract the costs from one and risks from one and then weight and add them to the weighted benefits and opportunities. In all, we have four different formulas for synthesis.

12. Perform sensitivity analysis on the final outcome and interpret the results of sensitivity by noting how stable this outcome is. Compare it with the other outcomes by taking ratios and observing how large or small these ratios are. Can another outcome that is close also serve as a best outcome? Why?

TWO WAYS TO FRAME THE QUESTION WHEN MAKING ANP COMPARISONS

When making pairwise comparisons in an ANP model the questions are formulated in terms of dominance or influence. Given a parent element, which of two elements being compared with respect to it has greater influence (is more dominant) with respect to that parent element? Or, which is influenced more with respect to that parent element? You want to avoid changing perspective. For example, in comparing A to B with respect to a criterion, you ask whether the criterion influences A or B more. Then if for the next comparison involving A and C you ask whether A or C influences the criterion more, this would be a change in perspective that would undermine the whole exercise. You must keep in mind whether the influence is flowing from the parent element to the elements being compared, or the other way around.

Use one of the following two questions throughout an exercise:

1. Given a parent element and comparing elements A and B under it, which element has greater influence *on* the parent element?
2. Given a parent element and comparing elements A and B, which element is influenced more *by* the parent element?

SUPER DECISIONS SOFTWARE TUTORIAL

INSTALLING THE ANP SOFTWARE

An alpha copy of the ANP software for decision-making in complex situations with dependence and feedback is available over the Internet or on a CD. The ANP software is called *SuperDecisions* and you may download it from <http://www.superdecisions.com>.

When you reach the home page, you must first register by selecting and entering a user name and password for yourself, then entering your name and entering your e-mail address. After registering you may get the ANP software. Click on download in the GET IT! Area. You must indicate that you agree with the software licensing policy. Then enter the operating system you have and select Alpha version to download (Beta is not available). You may download it either as a single file or as the two-file version that will fit on two disks. This software will time out after three months; but you may return to the www.superdecisions.com home page and download a current version. *If you forget your password you may send an email to creativdecisions.com requesting to have it sent to your email address.*

If you download the ANP software as a single large file (approximately 2.75 MB in size) it is labeled with the extension.exe. If you download the disk version it will be two smaller files (about 1.4 MB each) labeled with the extensions.exe and .w02 Download the file(s) into a temporary subdirectory, and follow the instructions below. If you have the software on a CD, or on 3 ½" diskettes, you may also install it by following the instructions below.

Go to Windows Explorer on your computer and get to the temp directory (if you downloaded from the internet), or your CD drive if you have the program on a CD, or your a: drive if you are using diskettes. Run the .exe file to install the *SuperDecisions* software. Do NOT click in the field when the second name with the .w02 extension appears, simply click on OK. Some internet explorers have a bad habit of adding a second dot before the .w02 if you click in the name field when the program pauses between the two files. This will cause it not to install correctly. If this happens, rename the file to get rid of the extra dot and try again. (For help with the software, you may send an e-mail to: rozann@creativedecisions.net)

INTRODUCTION TO THE SUPER DECISIONS SOFTWARE

In this introduction we review the ANP process and the *SuperDecisions* software and show some applications. Applications may be simple, consisting of a single network, or complex, consisting of a main network and two or more layers of sub-networks. Each network and sub-network is created in its own window.

In practice an application consists of:

- **A Simple Network** – *HamburgerModel*- All the clusters and their nodes are in a single window. An example of a simple network would be a “market share” application such as the Hamburger model. The simple network itself is the decision network because it contains the cluster of nodes that serve as the alternatives of the decision. In a market share application they are the competitors for whom market share is being predicted; for example, McDonald’s, Burger King and Wendy’s.
- **A Two-level Network** - *Car Purchase BCR* -There is a top-level network with Merit nodes such as Benefits, Opportunities, Costs and Risks, each of which has a sub-network. The alternatives cluster is in each of the sub-networks. The sub-networks are the decision networks because they contain the alternatives.
- **A Complex Network** - *National Missile Defense Model* -There is a main network of Merits nodes (Benefits, Opportunities, Costs and Risks), each having an attached sub-network that contains among others nodes that will serve as control criteria. The nodes selected to serve as control criteria, the high priority nodes in the network, have decision networks containing the alternatives attached to them. In practice this is the most complex system we work with, though there is no theoretical limitation on the number of levels of sub-networks.

A model is contained in a network system that physically is a single file. All the networks and sub-networks are in the same file. The file has the extension .sdmod; for example, the *Hamburger* model in Help>Sample Models>Introductory models is in the file named *Hamburger.sdmod*. If you have the *SuperDecisions* software installed an application can be launched by double-clicking on it within Windows Explorer. However, the files for the Encyclicon applications have been saved in an advanced zipped format to reduce their size. These files end with the .mod.gz extension. Such a zipped file cannot be launched by double-clicking. Instead you must open the *SuperDecisions* software first, then open the file using the File Open command.

There are four applications that we will discuss in the introduction: *Bridge*, *Hamburger*, *Car Purchase BCR*, and *National Missile Defense*. The *Bridge* example is a simple network used to illustrate the idea of feedback. The *Hamburger* example is a simple network used to estimate market

share of fast food restaurants. The *Car Purchase BCR* model is a two-level network with Benefits, Costs and Risks nodes in the top-level network, and the alternatives in the sub-networks attached to them. They range from the simplest example of feedback to a complex multi-level network structure. All of these examples are included in the sample models of the *SuperDecisions* software. To load a sample model use the Help, Sample Models command in the software and select the one you want. Sample models are located in the c:/program files/super decisions/samples directory.

Bridge is the simplest example in a single network and we use it to demonstrate the supermatrix idea.

Hamburger is used to estimate the market share of three fast-food hamburger places. The model consists of a single network containing the factors that consumers consider when choosing a fast-food restaurant. Some of its clusters are inner dependent with the nodes in them being compared with respect to other nodes in the same cluster. It has cluster comparisons as well as node comparisons. We use this model to explain more complicated supermatrices, inner and outer dependence and the motivation behind doing cluster comparisons. It is a simple network model in a single window.

Car Purchase BCR is a model for selecting the best kind of car to buy: Japanese, European or American, by taking into consideration the benefits, costs and risks of each type of car. It is a complex two-layer model with three sub-networks. The top-level network contains a benefits node, a costs node and a risks node, each of which has a sub-network where the alternatives are located. Judgments in a sub-network are made from the perspective of its controlling node in the network above.

National Missile Defense is the most complex kind of application with a top-level control model in which the priorities of the BOCR have been obtained by rating them against the US's national objectives. It has control criteria sub-networks under that and finally decision networks containing the alternatives at the bottom.

DEMONSTRATION OF THE SIMPLEST TYPE OF FEEDBACK NETWORK, THE BRIDGE MODEL

Bridge is a decision problem to pick the best of two bridge designs. It is a simple network of one level that contains only two clusters, with two nodes in each cluster, and links between the nodes. A network is structured of clusters, nodes and links. We use this model to show how feedback arises in a network decision structure and how the pairwise comparison questions are formulated when there is feedback. Here the clusters are outer dependent, that is, nodes in a cluster are compared only with respect to nodes in the other cluster.

Load the *Bridge* model by selecting Help, Sample Models from the main menu and selecting *bridge.sdm*

The *Bridge* model, a simple network model, is shown in Figure 15. Clusters may be re-sized by left-clicking on the small button at the lower right hand corner and dragging. To select a cluster left-click on the title bar. A cluster is selected when its title bar is highlighted. Left-click on the title bar of a cluster window and drag to move it to a different location.

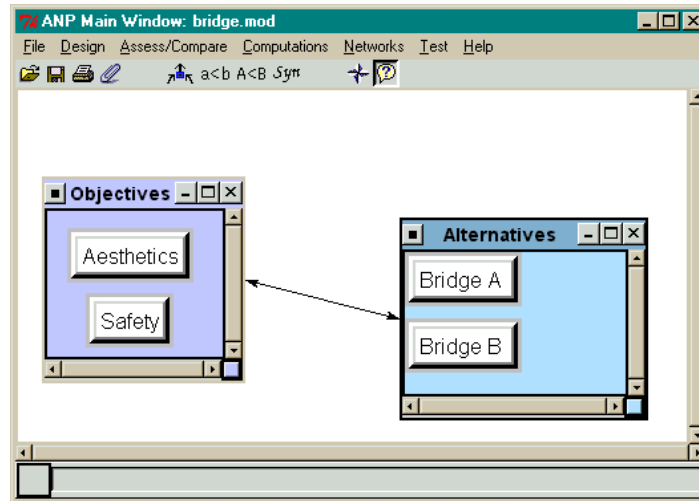



Figure 15. The *Bridge Model*: a Simple Network.

The decision in this model is to select the best bridge. The objectives are to have a safe bridge and an aesthetically pleasing bridge. If one were doing the model from the top down as in a hierarchy, there would be a goal with Aesthetics and Safety as the criteria and Bridge A and Bridge B as the alternatives. One would compare Aesthetics to Safety, Safety would likely be perceived as extremely more important, so the safest bridge would be the "best" choice.

But in a feedback network one compares the bridges for preference with respect to Aesthetics and to Safety, and one also compares the prevalence of Aesthetics versus Safety for each bridge. The net result of this is that priorities are obtained for all four nodes in the system. Suppose the safest bridge, B, is unattractive, and the nicer looking bridge, A, is very safe, though not as safe as B. The priorities of the criteria depend on the bridges available and since both are quite safe, the priority of safety in the feedback system ends up less than it would be in a hierarchy where one compares Safety to Aesthetics in the abstract and apart from any specific bridge. It makes common sense that if both bridges are very safe, one should pick the better-looking bridge, even though one bridge is far safer than the other. The Analytic Network Process through feedback guides us to the best choice in a way that matches our common sense.

The Aesthetics node in the Objectives cluster is linked to Bridge A and to Bridge B, and because there is a link between nodes, a link appears from the Objectives cluster to the Alternatives cluster. Because at least one node in the Objectives cluster is linked to nodes in the Alternatives cluster, a link appears automatically from the Objectives cluster to the Alternatives cluster. Aesthetics is the parent node and Bridge A will be compared to Bridge B with respect to it. The node Safety is also linked to Bridge A and Bridge B, and they will be compared for preference with respect to safety.

To turn on the "show connections" mode, as shown in Figure 16 click on the star-shaped icon . When you place your cursor over a node when this icon is depressed, the nodes connected from it will be outlined in red. Try this by placing the cursor over the Bridge A node and the Aesthetics and Safety nodes will be outlined in red.

When a node has had the comparisons marked as completed for nodes within a cluster that are connected to it, the cluster window of these nodes will also be outlined in red. Both bridges are also connected to the nodes in the Objectives cluster, so holding the cursor over the Bridge A node will show Aesthetics

and Safety outlined in red, and the Objectives cluster being outlined in red indicates that the comparison of these nodes with respect to Bridge A is complete.

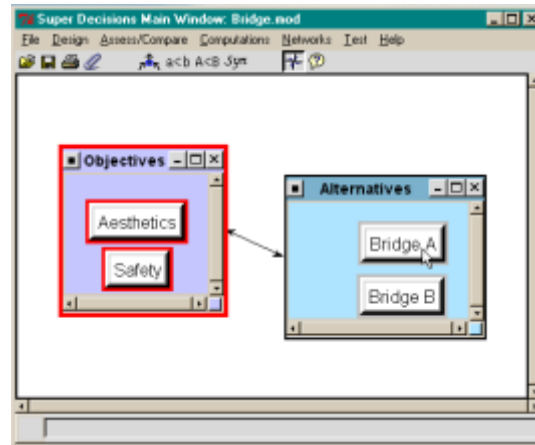


Figure 16. The Aesthetics Node is connected to Bridge A and Bridge B.

FEEDBACK LINKS

It is easy for those who have used the Analytic Hierarchy Process to understand how to pairwise compare Bridge A and Bridge B with respect to Aesthetics. Bridge A would be highly preferred. But what may be new is the idea that criteria may be compared with respect to an alternative. What does that mean? When comparing, for example, Aesthetics and Safety with respect to Bridge A, the question is: What is more a more pronounced or prevalent characteristic of Bridge A, its aesthetics or its safety? Bridge A is beautiful and that is what we like best about it. Its safety, though quite adequate, is nothing notable. So we strongly prefer its aesthetics to its safety.

For Bridge B what is its more preferable characteristic, aesthetics or safety? Since it is quite ugly, the answer is that the Safety of Bridge B is extremely preferable to its Aesthetics. These kinds of preference questions and answers, both directions, help us establish our true priorities for *all* the elements in the problem.

THE SUPERMATRIX

The comparison process will be covered in the next demonstration. Here we will show the various computations involving the supermatrix. To show the three different supermatrices, select the Computations command from the menu shown in Figure 17.

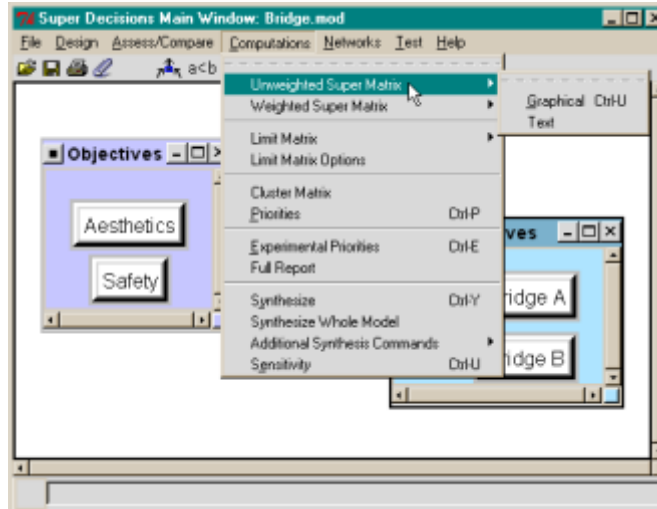


Figure 17. The Computations Menu.

THE UN-WEIGHTED, WEIGHTED AND LIMIT SUPERMATRICES

There are three supermatrices associated with each network: the Unweighted Supermatrix, the Weighted Supermatrix and the Limit Supermatrix. Supermatrices are arranged with the clusters in alphabetical order across the top and down the left side, and with the elements within each cluster in alphabetical order across the top and down the left side. To change the ordering in a supermatrix, you need only re-name the clusters and/or the elements, so their alphabetical order will be the order you want. Changing names after building a model and making comparisons is permitted and will correctly preserve any judgments that have been made.

The unweighted supermatrix contains the local priorities derived from the pairwise comparisons throughout the network as shown in Figure 18. For example, the priorities of the elements Aesthetics and Safety, with respect to Bridge A are shown in the two bottom cells of the first column, 0.857143 and 0.142857. This may be interpreted with the statement, "The Aesthetics characteristic of Bridge A is between strongly and very strongly, or 6 times, more its dominant preferred characteristic than its Safety aspect." All the local priority information can be read directly from the unweighted Supermatrix.

Cluster Node Labels		Alternatives		Objectives	
		Bridge A	Bridge B	Aesthetic s	Safety
Alternati ves	Bridge A	0.000000	0.000000	0.875000	0.333333
	Bridge B	0.000000	0.000000	0.125000	0.666667
Objectiv es	Aesthetic s	0.857143	0.100000	0.000000	0.000000
	Safety	0.142857	0.900000	0.000000	0.000000
Done					

Figure 18. The Unweighted Supermatrix for the Bridge Model.

Definition of Component: A component in a supermatrix is the block defined by a cluster name at the left and a cluster name at the top. For example, the (Alternatives, Alternatives) component in Figure 18 is composed of the block of four zeros in the upper left-hand corner shown in the screen clip below.

Detail of (Alternatives, Alternatives) Component from Figure 18

Cluster Node Labels		Alternatives	
		Bridge A	Bridge B
Alternatives	Bridge A	0.000000	0.000000
	Bridge B	0.000000	0.000000

The (Alternatives, Objectives) component is the block of four numbers in the left hand bottom corner of Figure 18 as shown in the screen clip below:

Detail of (Objectives, Alternatives) Component from Figure 18

Objectives	Aesthetics	0.857143	0.100000
	Safety	0.142857	0.900000

The weighted supermatrix is obtained by multiplying all the elements in a component of the unweighted supermatrix by the corresponding cluster weight. We will say more about cluster weights when we demonstrate the *Hamburger* model later. In this example there were no cluster comparisons, since there are only two clusters and cluster comparisons cannot be made when there are only two. The weighted and unweighted supermatrices are the same in this example and are shown in Figure 19. Notice that as the columns already sum to one in the unweighted supermatrix there is no need to weight the components to make the columns sum to one in the weighted supermatrix.

ANP Main Window: bridge.mod: Weighted Super Ma...					
Cluster Node Labels		Alternatives		Objectives	
		Bridge A	Bridge B	Aesthetics	Safety
Alternatives	Bridge A	0.000000	0.000000	0.875000	0.333333
	Bridge B	0.000000	0.000000	0.125000	0.666667
Objectives	Aesthetics	0.857143	0.100000	0.000000	0.000000
	Safety	0.142857	0.900000	0.000000	0.000000
Done					

Figure 19. The Weighted (same as Unweighted) Supermatrix for the Bridge Model.

The limit supermatrix is obtained by raising the weighted supermatrix to powers by multiplying it times itself. When the column of numbers is the same for every column, the limit matrix has been reached and the matrix multiplication process is halted. The limit supermatrix for the *Bridge* Model is shown in Figure 20.

Cluster Node Labels		Alternatives		Objectives	
		Bridge A	Bridge B	Aesthetics	Safety
Alternatives	Bridge A	0.328456	0.328456	0.328456	0.328456
	Bridge B	0.171544	0.171544	0.171544	0.171544
Objectives	Aesthetics	0.298688	0.298688	0.298688	0.298688
	Safety	0.201312	0.201312	0.201312	0.201312

Done

Figure 20. The Limit Supermatrix for the Bridge Model.

The Computations Priorities command on the menu displays the priorities in two ways: as they appear in the supermatrix, and with the priorities normalized by cluster as shown in Figure 21. The columns of the limit supermatrix are all the same, so the priorities for all the nodes can be read from any column.

Icon	Name	Normalized by Cluster	Limiting
No Icon	Bridge A	0.65691	0.328456
No Icon	Bridge B	0.34309	0.171544
No Icon	Aesthetics	0.59738	0.298688
No Icon	Safety	0.40262	0.201312

Okay

Figure 21. The Priorities from the Limit Supermatrix

The Computations Synthesize command displays the final results in three ways as shown in Figure 22. The *Raw* column gives the priorities from the limiting supermatrix (which also appear in the *Limiting* column above), the *Normals* column shows the results normalized for each component (which also appear in the *Normalized by Cluster* column above) and the *Ideals* column shows the results obtained by dividing the values in either the normalized or limiting columns by the largest value in the column.

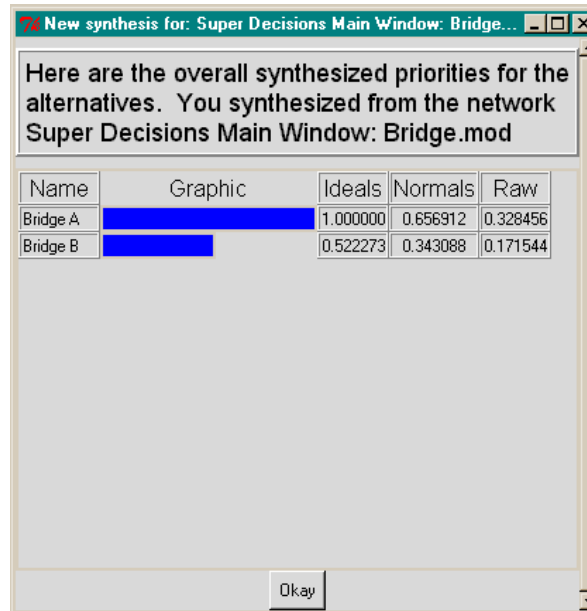


Figure 22. The Synthesized Values – the Results for the Alternatives.

The results show that Bridge A is best. This matches our intuition: "Bridge A is very safe and also the best looking, so choose it. Bridge B has overkill so far as safety is concerned, and it is not good looking – so it is not the best bridge overall. Do not choose it."

DEMO OF THE HAMBURGER MODEL - A SIMPLE NETWORK MODEL

The purpose of the *Hamburger* model, a simple network application shown in Figure 23, is to estimate the market share of three fast-food hamburger joints. A simple network has all the clusters and their nodes in a single window. There are no sub-networks. All the comparison questions are asked from the perspective of what is more important or preferred with respect to market share. In this demo we will explain the motivation behind doing cluster comparisons.

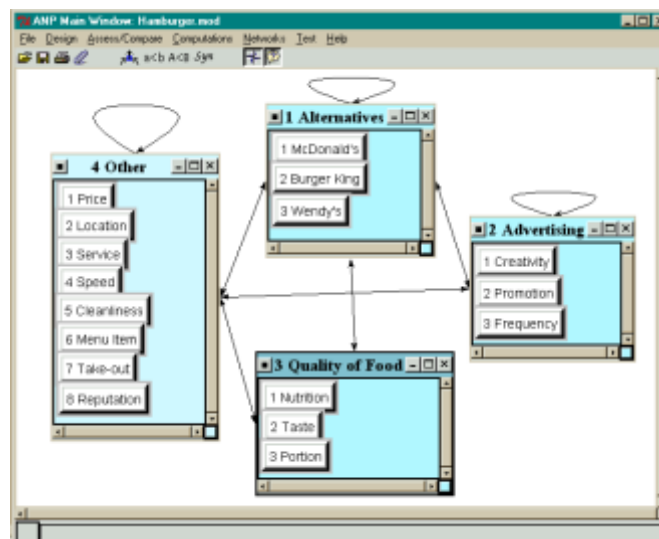


Figure 23. The *Hamburger* Model for Predicting Market Share.

In this model the loops indicate inner dependence among the elements in the cluster. In Figure 24 is a view of the model with icons instead of cluster windows. To switch back and forth from a cluster icon to a cluster window double left click with your mouse on it.

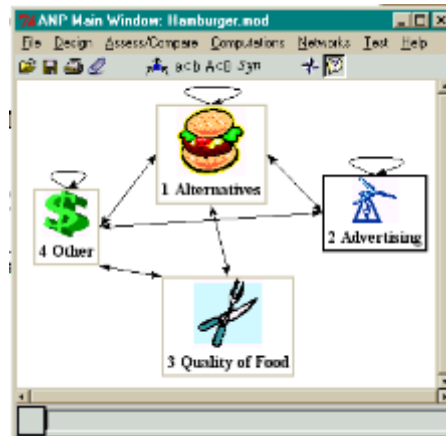


Figure 24. An Iconized View of the Hamburger Model.

Shortcut to Cluster menu: Right click on the background within any cluster to get a drop-down menu of cluster commands as shown in Figure 25 below.

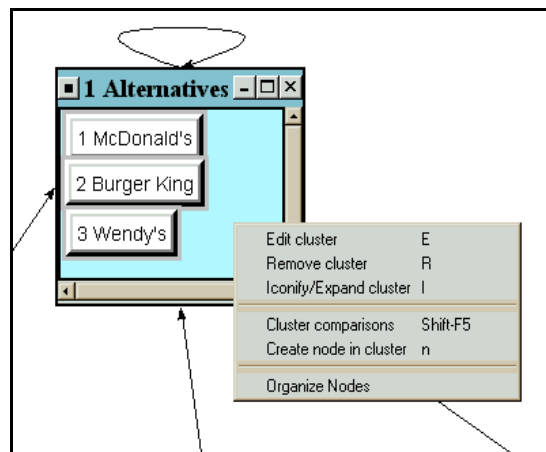


Figure 25. Drop-down Menu of Cluster Commands.

Select the Edit cluster command to change fonts, choose colors or icons, or type definitions. The Organize Nodes command can be used to find missing nodes that may have accidentally scrolled off the window.

COMPARISONS

Pairwise comparisons for the nodes in each cluster that belong to a parent node are carried out for all the parent nodes in the model. Select the Assess/Compare command, then select the cluster and the node to serve as the parent node. Select the cluster containing nodes to be compared with respect to the parent node. You can also start the comparison process by using the drop down node menu.

Shortcut to Node menu: Right click on a node, McDonald's, for instance, for the drop down menu with commands relating to that node to appear as shown in Figure 26.

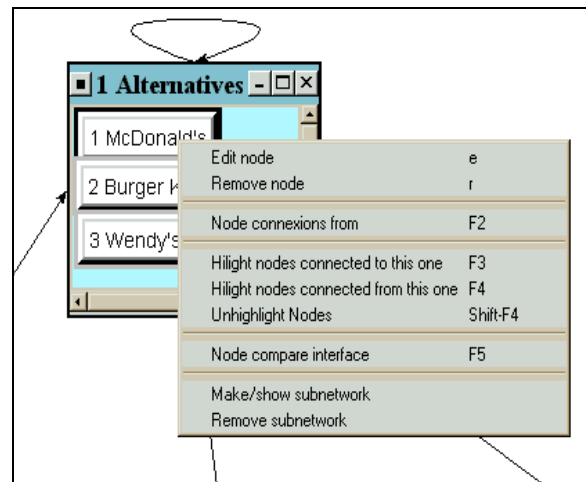


Figure 26. The Dropdown Menu for a Node.

To initiate comparisons with respect to a selected node select the Node Compare Interface command from the drop-down menu, then select the cluster having the nodes you want to compare with respect to McDonald's. This will bring up the comparisons screen in the Questionnaire mode as shown in Figure 27. You can select from several ways to do comparisons: Graphic, Verbal, Matrix, and Questionnaire listed on the tabs at the top.

Figure 27. The Questionnaire Mode for Comparing Nodes in the "Other" Cluster with respect to McDonald's

To switch from the Questionnaire mode to the Matrix mode shown in Figure 28 click on the Matrix tab.

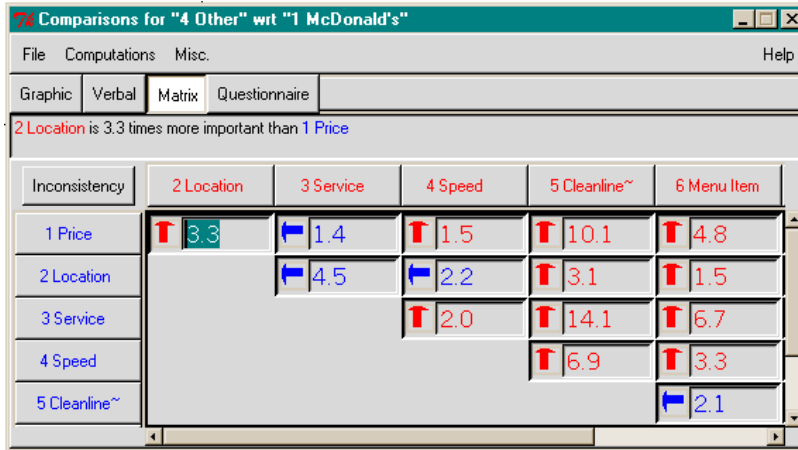


Figure 28. The Matrix Mode for Comparing Nodes in the “Other” Cluster with respect to McDonald’s

To show all the judgments in the matrix you must use the scroll bars at the right side and bottom of the window. A judgment is entered in each cell. A cell contains the comparison for the pair listed at the top and at the side. The arrows in the Matrix mode point toward the preferred node of the pair. The top node is preferred when the arrow is red, the side node when the arrow is blue. To toggle a comparison between red and blue, double-click on the arrow button. This inverts the comparison so that the other node is preferred.

Tip: To invert a judgment, double-click on the arrow button associated with it.

To compute the local priorities associated with these judgments, select the Computations, Show New Priorities command. The priorities of the nodes in the Other cluster with respect to McDonald's will be displayed as shown in Figure 29. Consistency can also be improved from the Computations menu. Select the Okay bar at the bottom of the window to return to the Comparison mode. Select File, Save Changes and File, Close to return to the main view.

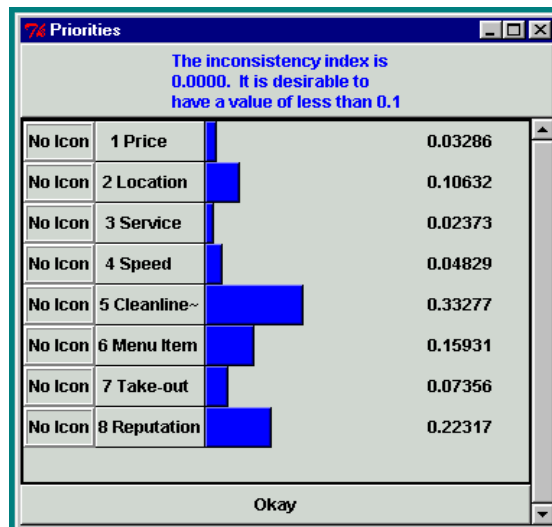


Figure 29. The Local Priorities for Nodes in "Other", Compared with Respect to McDonald's.

The results of all the pairwise comparisons are entered in the unweighted supermatrix. To display it select Computations, Unweighted Supermatrix. To print any of the supermatrices select the command File, Export, and type of supermatrix to be exported, for example, select Unweighted to export to the file

hamburger.unweighted.txt. This file can then be imported into Excel to format and print as you like by opening Excel and selecting File, Import and enter the name of the text file that was exported. The Excel Text Import Wizard will appear and help you through the process. Select Delimited, then select Tab with the Text Qualifier field set to ", accept the General Data format for columns and finish. The unweighted supermatrix will then be loaded into Excel and will include row and column headings of both clusters and nodes.

Examples are shown below of such imported tables. They were obtained by first exporting from the *SuperDecisions* program, then importing to Excel, formatting within Excel, then importing into Word. The first table gives the cluster weights matrix. The values in the cluster matrix are used to weight the unweighted supermatrix by multiplying the value in the (Alternatives, Alternatives) cell of the cluster matrix times the value in each cell in the (Alternatives, Alternatives) component of the unweighted supermatrix to produce the weighted supermatrix. Every component is weighted with its corresponding Cluster Matrix weight in this way.

Table 17. Cluster Weights Matrix

	Alternatives	Advertising	Quality	Other
1. Alternatives	0.2128	0.2956	0.5000	0.1304
2. Advertising	0.5319	0.2571	0.0000	0.6079
3. Quality	0.0659	0.0000	0.0000	0.0655
4. Other	0.1893	0.4473	0.5000	0.1969

Table 18. Supermatrix of Unweighted Priorities (shown in two parts)

		1 Alternatives			2 Advertising			3 Quality of Food		
		1 McDonald's	2 Burger King	3 Wendy's	1 Creativity	2 Promotion	3 Frequency	1 Nutrition	2 Taste	3 Portion
1 Alternati~	1 McDonald's	0.0000	0.8333	0.7500	0.6141	0.7174	0.7174	0.2488	0.2899	0.5989
	2 Burger King	0.8000	0.0000	0.2500	0.2685	0.1942	0.1942	0.1561	0.1040	0.1262
	3 Wendy's	0.2000	0.1667	0.0000	0.1174	0.0884	0.0884	0.5951	0.6061	0.2749
2 Advertisi~	1 Creativity	0.2074	0.1783	0.2810	0.0000	0.3333	0.5000	0.0000	0.0000	0.0000
	2 Promotion	0.1298	0.1120	0.0720	0.1250	0.0000	0.5000	0.0000	0.0000	0.0000
	3 Frequency	0.6628	0.7096	0.6470	0.8750	0.6667	0.0000	0.0000	0.0000	0.0000
3 Quality o~	1 Nutrition	0.3319	0.2810	0.6241	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2 Taste	0.1388	0.0720	0.2823	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3 Portion	0.5293	0.6470	0.0936	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4 Other	1 Price	0.0329	0.2408	0.0300	0.0000	0.8333	0.0000	0.0000	0.0000	0.8571
	2 Location	0.1063	0.2231	0.1417	0.7095	0.0000	0.1958	0.0000	0.0000	0.0000
	3 Service	0.0237	0.1418	0.0648	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4 Speed	0.0483	0.1407	0.0641	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5 Cleanliness	0.3328	0.1096	0.2756	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6 Menu Item	0.1593	0.0512	0.1571	0.1377	0.1667	0.3108	0.0000	0.0000	0.0000
	7 Take-out	0.0736	0.0506	0.0589	0.0000	0.0000	0.0000	0.0000	0.0000	0.1429
	8 Reputation	0.2232	0.0422	0.2078	0.1528	0.0000	0.4934	0.0000	0.0000	0.0000

...

		4 Other							
		1 Price	2 Location	3 Service	4 Speed	5 Cleanliness	6 Menu Item	7 Take-out	8 Reputation
1 Alternati~	1 McDonald's	0.6531	0.6531	0.3319	0.5387	0.2500	0.4934	0.4837	0.6749
	2 Burger Ki~	0.2507	0.2507	0.1388	0.3624	0.2500	0.1958	0.3133	0.2238
	3 Wendy's	0.0962	0.0962	0.5293	0.0989	0.5000	0.3108	0.2029	0.1012
2 Advertisi~	1 Creativity	0.0000	0.0000	0.0000	0.0000	0.0000	0.0780	0.0000	0.0819
	2 Promotion	0.8333	0.0000	0.0000	0.0000	0.0000	0.1711	0.0000	0.3678
	3 Frequency	0.1667	0.0000	0.0000	0.0000	0.0000	0.7509	0.0000	0.5503
3 Quality o~	1 Nutrition	0.1667	0.0000	0.0000	0.0000	0.0000	0.0756	0.0000	0.0936
	2 Taste	0.0000	0.0000	0.0000	0.0000	0.0000	0.6952	0.0000	0.6241
	3 Portion	0.8333	0.0000	0.0000	0.0000	0.0000	0.2292	0.0000	0.2823
4 Other	1 Price	0.0000	0.0000	0.0000	0.0000	0.0000	0.1153	0.0000	0.0627
	2 Location	0.5000	0.0000	0.0981	0.0000	0.1711	0.0526	0.6572	0.2653
	3 Service	0.0000	0.0000	0.0000	0.1873	0.0780	0.0000	0.0548	0.0444
	4 Speed	0.0000	0.0000	0.2857	0.0000	0.7509	0.1946	0.2880	0.0835
	5 Cleanline~	0.0000	0.0000	0.5181	0.0000	0.0000	0.6375	0.0000	0.2378
	6 Menu Item	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1929
	7 Take-out	0.5000	0.0000	0.0000	0.7313	0.0000	0.0000	0.0000	0.0567
	8 Reputation	0.0000	0.0000	0.0981	0.0814	0.0000	0.0000	0.0000	0.0567

Table 19. Weighted Supermatrix

		1 Alternatives			2 Advertising			3 Quality of food		
		1 McDonald's	2 Burger King	3 Wendy's	1 Creativity	2 Promotion	3 Frequency	1 Nutrition	2 Taste	3 Portion
1 Alternati~	1 McDonald's	0.0000	0.1774	0.1596	0.1815	0.2121	0.2121	0.2488	0.2899	0.2995
	2 Burger Ki~	0.1703	0.0000	0.0532	0.0794	0.0574	0.0574	0.1561	0.1040	0.0631
	3 Wendy's	0.0426	0.0355	0.0000	0.0347	0.0261	0.0261	0.5951	0.6061	0.1375
2 Advertisi~	1 Creativity	0.1103	0.0949	0.1495	0.0000	0.0857	0.1286	0.0000	0.0000	0.0000
	2 Promotion	0.0690	0.0596	0.0383	0.0321	0.0000	0.1286	0.0000	0.0000	0.0000
	3 Frequency	0.3526	0.3775	0.3442	0.2250	0.1714	0.0000	0.0000	0.0000	0.0000
3 Quality o~	1 Nutrition	0.0219	0.0185	0.0411	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2 Taste	0.0091	0.0047	0.0186	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3 Portion	0.0349	0.0427	0.0062	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4 Other	1 Price	0.0062	0.0456	0.0057	0.0000	0.3727	0.0000	0.0000	0.0000	0.4286
	2 Location	0.0201	0.0422	0.0268	0.3173	0.0000	0.0876	0.0000	0.0000	0.0000
	3 Service	0.0045	0.0268	0.0123	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4 Speed	0.0091	0.0266	0.0121	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5 Cleanline~	0.0630	0.0207	0.0522	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6 Menu Item	0.0302	0.0097	0.0297	0.0616	0.0745	0.1390	0.0000	0.0000	0.0000
	7 Take-out	0.0139	0.0096	0.0112	0.0000	0.0000	0.0000	0.0000	0.0000	0.0714
	8 Reputation	0.0422	0.0080	0.0393	0.0684	0.0000	0.2207	0.0000	0.0000	0.0000

...

		4 Other							
		1 Price	2 Location	3 Service	4 Speed	5 Cleanliness	6 Menu Item	7 Take-out	8 Reputation
1 Alternati~	1 McDonald's	0.0852	0.6531	0.1326	0.2151	0.0998	0.0643	0.1932	0.0880
	2 Burger Ki~	0.0327	0.2507	0.0554	0.1447	0.0998	0.0255	0.1251	0.0292
	3 Wendy's	0.0125	0.0962	0.2114	0.0395	0.1997	0.0405	0.0810	0.0132
2 Advertisi~	1 Creativity	0.0000	0.0000	0.0000	0.0000	0.0000	0.0474	0.0000	0.0498
	2 Promotion	0.5066	0.0000	0.0000	0.0000	0.0000	0.1040	0.0000	0.2236
	3 Frequency	0.1013	0.0000	0.0000	0.0000	0.0000	0.4565	0.0000	0.3345
3 Quality o~	1 Nutrition	0.0109	0.0000	0.0000	0.0000	0.0000	0.0050	0.0000	0.0061
	2 Taste	0.0000	0.0000	0.0000	0.0000	0.0000	0.0456	0.0000	0.0409
	3 Portion	0.0546	0.0000	0.0000	0.0000	0.0000	0.0150	0.0000	0.0185
4 Other	1 Price	0.0000	0.0000	0.0000	0.0000	0.0000	0.0226	0.0000	0.0123
	2 Location	0.0981	0.0000	0.0589	0.0000	0.1028	0.0103	0.3947	0.0520
	3 Service	0.0000	0.0000	0.0000	0.1125	0.0469	0.0000	0.0329	0.0087
	4 Speed	0.0000	0.0000	0.1716	0.0000	0.4510	0.0382	0.1730	0.0164
	5 Cleanline~	0.0000	0.0000	0.3112	0.0000	0.0000	0.1250	0.0000	0.0466
	6 Menu Item	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0378
	7 Take-out	0.0981	0.0000	0.0000	0.4393	0.0000	0.0000	0.0000	0.0111
	8 Reputation	0.0000	0.0000	0.0589	0.0489	0.0000	0.0000	0.0000	0.0111

Table 20. Limiting Supermatrix

		1 Alternatives			2 Advertising			3 Quality of food		
		1 McDonald's	2 Burger King	3 Wendy's	1 Creativity	2 Promotion	3 Frequency	1 Nutrition	2 Taste	3 Portion
1 Alternati~	1 McDonald's	0.1749	0.1749	0.1749	0.1749	0.1749	0.1749	0.1749	0.1749	0.1749
	2 Burger Ki~	0.0883	0.0883	0.0883	0.0883	0.0883	0.0883	0.0883	0.0883	0.0883
	3 Wendy's	0.0520	0.0520	0.0520	0.0520	0.0520	0.0520	0.0520	0.0520	0.0520
2 Advertisi~	1 Creativity	0.0727	0.0727	0.0727	0.0727	0.0727	0.0727	0.0727	0.0727	0.0727
	2 Promotion	0.0878	0.0878	0.0878	0.0878	0.0878	0.0878	0.0878	0.0878	0.0878
	3 Frequency	0.1905	0.1905	0.1905	0.1905	0.1905	0.1905	0.1905	0.1905	0.1905
3 Quality o~	1 Nutrition	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087
	2 Taste	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076
	3 Portion	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145
4 Other	1 Price	0.0462	0.0462	0.0462	0.0462	0.0462	0.0462	0.0462	0.0462	0.0462
	2 Location	0.0681	0.0681	0.0681	0.0681	0.0681	0.0681	0.0681	0.0681	0.0681
	3 Service	0.0091	0.0091	0.0091	0.0091	0.0091	0.0091	0.0091	0.0091	0.0091
	4 Speed	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248
	5 Cleanline~	0.0271	0.0271	0.0271	0.0271	0.0271	0.0271	0.0271	0.0271	0.0271
	6 Menu Item	0.0474	0.0474	0.0474	0.0474	0.0474	0.0474	0.0474	0.0474	0.0474
	7 Take-out	0.0210	0.0210	0.0210	0.0210	0.0210	0.0210	0.0210	0.0210	0.0210
	8 Reputation	0.0596	0.0596	0.0596	0.0596	0.0596	0.0596	0.0596	0.0596	0.0596

...

		4 Other							
		1 Price	2 Location	3 Service	4 Speed	5 Cleanliness	6 Menu Item	7 Take-out	8 Reputation
1 Alternati~	1 McDonald's	0.1749	0.1749	0.1749	0.1749	0.1749	0.1749	0.1749	0.1749
	2 Burger Ki~	0.0883	0.0883	0.0883	0.0883	0.0883	0.0883	0.0883	0.0883
	3 Wendy's	0.0520	0.0520	0.0520	0.0520	0.0520	0.0520	0.0520	0.0520
2 Advertisi~	1 Creativity	0.0727	0.0727	0.0727	0.0727	0.0727	0.0727	0.0727	0.0727
	2 Promotion	0.0878	0.0878	0.0878	0.0878	0.0878	0.0878	0.0878	0.0878
	3 Frequency	0.1905	0.1905	0.1905	0.1905	0.1905	0.1905	0.1905	0.1905
3 Quality o~	1 Nutrition	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087
	2 Taste	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076
	3 Portion	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145
4 Other	1 Price	0.0462	0.0462	0.0462	0.0462	0.0462	0.0462	0.0462	0.0462
	2 Location	0.0681	0.0681	0.0681	0.0681	0.0681	0.0681	0.0681	0.0681
	3 Service	0.0091	0.0091	0.0091	0.0091	0.0091	0.0091	0.0091	0.0091
	4 Speed	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248
	5 Cleanline~	0.0271	0.0271	0.0271	0.0271	0.0271	0.0271	0.0271	0.0271
	6 Menu Item	0.0474	0.0474	0.0474	0.0474	0.0474	0.0474	0.0474	0.0474
	7 Take-out	0.0210	0.0210	0.0210	0.0210	0.0210	0.0210	0.0210	0.0210
	8 Reputation	0.0596	0.0596	0.0596	0.0596	0.0596	0.0596	0.0596	0.0596

Table 21. Limiting Priorities and Normalized by Cluster Priorities

		Priorities from Limiting Matrix	Priorities Normalized by Cluster
1 Alternatives	1 McDonald's	0.1749	0.5549
	2 Burger King	0.0883	0.2801
	3 Wendy's	0.0520	0.1650
2 Advertising	1 Creativity	0.0727	0.2071
	2 Promotion	0.0878	0.2501
	3 Frequency	0.1905	0.5427
3 Quality of food	1 Nutrition	0.0087	0.2825
	2 Taste	0.0076	0.2468
	3 Portion	0.0145	0.4708
4 Other	1 Price	0.0462	0.1523
	2 Location	0.0681	0.2245
	3 Service	0.0091	0.0300
	4 Speed	0.0248	0.0818
	5 Cleanliness	0.0271	0.0894
	6 Menu Item	0.0474	0.1563
	7 Take-out	0.0210	0.0692
	8 Reputation	0.0596	0.1965

MAKING CLUSTER COMPARISONS

To compare clusters take each cluster in turn (as the parent) and pairwise compare all the clusters it connects to for importance with respect to their influence on it. This is how the Cluster Matrix is generated. Keep in mind that the overall goal here is Market Share. For example, select Assess, Compare, Cluster comparisons and choose the Alternatives cluster. The comparison process now is used to pairwise compare the clusters for influence to which the parent cluster connects. Here is an example of how you formulate a cluster comparison question: "Which influences the market share of the Alternatives more, Advertising or Quality of Food?" The judgment is made that advertising is between very strongly and extremely more important than the quality of food in influencing market share, and a value of 8.1 is entered.

MAKING INNER DEPENDENT CLUSTER COMPARISONS

As the Alternatives cluster is inner dependent, it is connected to itself, so it is one of the four clusters being pairwise compared with respect to Alternatives. The only time you have to ask such a question, comparing a cluster against another cluster with respect to itself as the parent cluster, is when it exhibits inner dependence. Since nodes in the Alternatives cluster are connected to other nodes in that cluster, it must influence itself. If it makes sense to ask the question: "Does Wendy's or Burger King influence McDonald's market share more?" so it needs to be compared for how important its influence is on itself compared to the other clusters it links to. The Wendy's versus Burger King question makes sense. We are really asking which one is a stronger competitor of McDonald's. A judgment of 4 was entered for Burger King over Wendy's; that is, Burger King is a stronger competitor, and that seems reasonable as Burger King has the second largest market share of the three alternatives.

WHY MAKE CLUSTER COMPARISONS?

If all the clusters are equally important it is not necessary to make cluster comparisons, and the cluster weights are set to $1/n$ in the cluster matrix. The value of n is equal to the number of non-zero components beneath each component across the top of the unweighted supermatrix.

However, the clusters in a network may not be equally important. Then they need to be compared to establish the weights in the cluster matrix. Weighting all the elements in each unweighted supermatrix component by the corresponding cluster matrix cell, whether set by the default value of $1/n$ described above, or by comparing the clusters and using the derived values, causes the matrix to be column stochastic, that is, each column sums to one. In Table 2 for the unweighted supermatrix the first column sums to 4.000. In Table 3 for the weighted supermatrix it sums to 1.000.

It is essential in real life problems that one know the importance of the groups or clusters to which the elements belong because the final priorities do (and should) depend on that. For example, a society of astronomers is not as important to immediate human survival as the society of farmers, although on the face of it an astronomer may seem more important than a farmer because there are a much smaller number of them (and some people say the greatest potential disaster is a comet crashing into the earth – so astronomers might help avoid that).

Select the Computations, Cluster Matrix command to display the Cluster matrix for the *Hamburger* model shown in Figure 30.

Cluster Node Labels	1 Alternatives	2 Advertising	3 Quality of Food	4 Other
1 Alternatives	0.212821	0.295623	0.500000	0.130395
2 Advertising	0.531948	0.257117	0.000000	0.607943
3 Quality of Food	0.065817	0.000000	0.000000	0.065549
4 Other	0.189315	0.447260	0.500000	0.196114

Figure 30. The Cluster Matrix

THE FINAL RESULTS

The final results for the Hamburger model are obtained by selecting the command Computations, Synthesize. The results are shown in Table 5: McDonald's has 55.49% of the market share, Burger King has 28.01% and Wendy's has 16.50%. At the time the model was done the actual values published in the *Market Share Reporter* of 1994 were: 58.23%, 28.57% and 13.20%.

DEMONSTRATION OF A TWO-LAYER SYSTEM, THE CAR PURCHASE BCR MODEL

This model will be used to show a 2-level model with a top-level control network and three sub-networks. It is a model to pick the best type of car: European, Japanese or American. The top-level control network is actually a hierarchy with three control criteria: Benefits, Costs and Risks, and there is a sub-network

associated with each. Every sub-network must contain a cluster with the alternatives in it, and these sub-networks do.

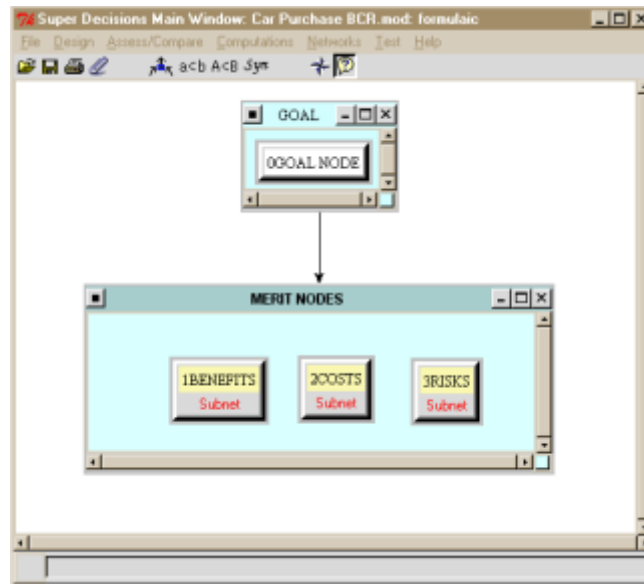


Figure 31. The Top Level Network for the *Car Purchase BCR* Model.

MAKE/SHOW SUBNETS

To make or show a sub-network for a node, right click on the background of the node and select the command Make/show sub-network as shown in the screen clip below. If there is an existing sub-network, it will open. If not, a blank window will appear. When a node has an attached sub-network, the word Subnet is tacked onto the node name. After creating a sub-network, you can open it by left double-clicking on the node name of the node it is attached to. You *must* click on the node name and *not* the red area with the word Subnet on it to bring up the sub-network.

NODE MENU

To have a drop-down node menu appear, right-click on any node, for example the Risks node, as shown in Figure 32 below. Commands invoked from this menu will apply to the Risk node.

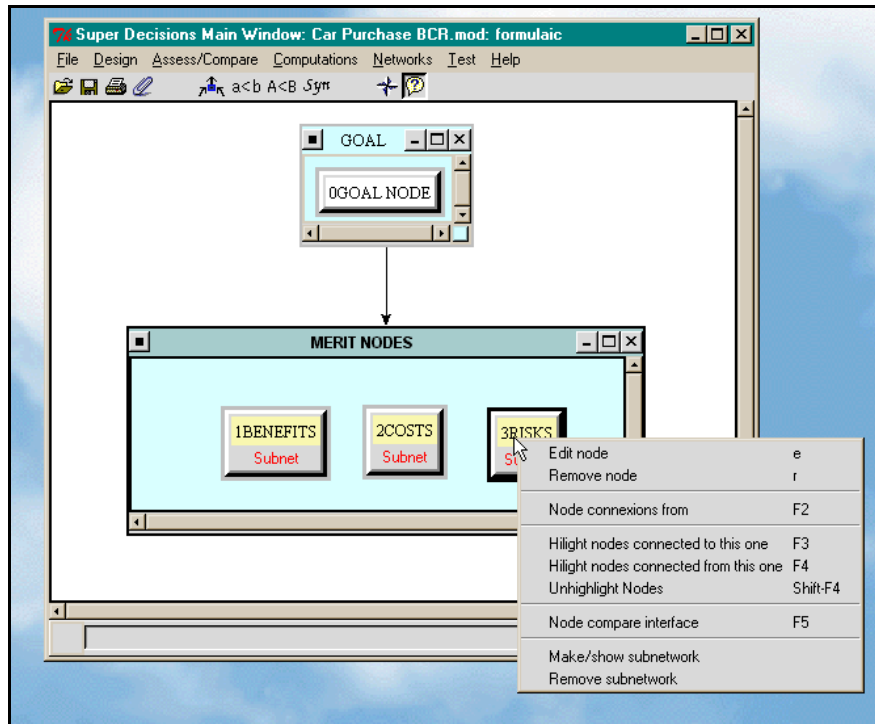


Figure 32. Node Menu Commands

The sub-networks are contained in separate Windows as shown in the view of the entire model in Figure 33 below. To open the subnet attached to the Benefits node, for example, click on the word Benefits (*not* on the red word Subnet beneath it!) Note that the subnets here are decision subnets as each has an Alternatives cluster in it. The command structure in a subnet window is the same as in the top-level network window.

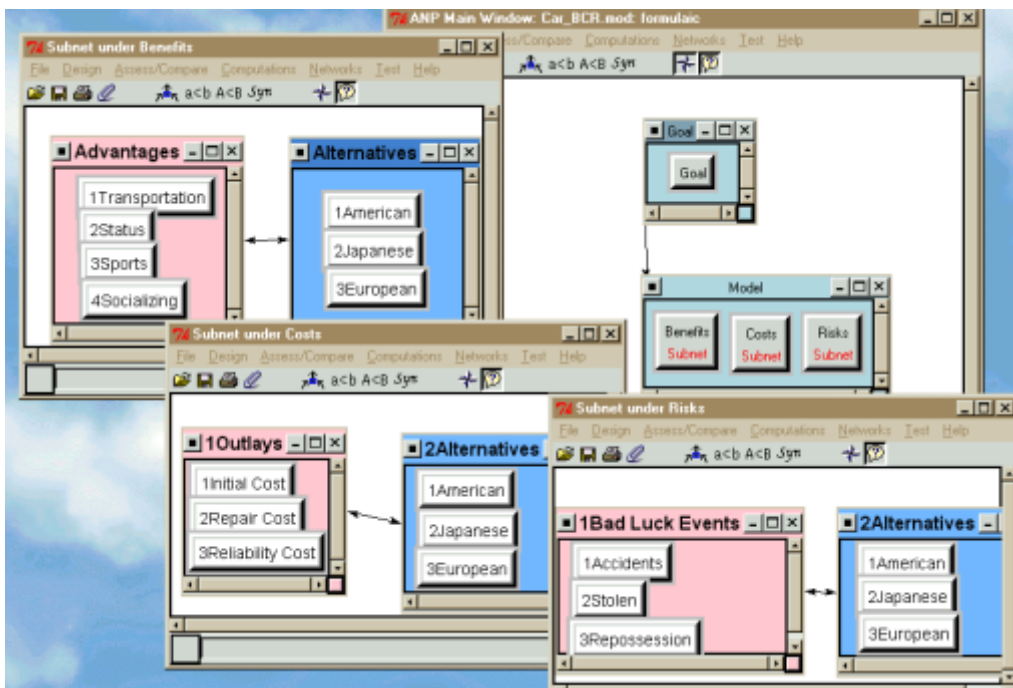


Figure 33. Car Purchase BCR Model View of All Networks.

Tip: In handling the subnet windows if one disappears it may have become minimized and be down on the taskbar at the bottom of the desktop screen. Left-click on it to restore it.

Tip on Saving: You can save a complete complex model by selecting the File, Save command from either the top-level network or from a subnet. You can also opt to save only a subnet. You can save subnets and use them as templates later by opening in the blank window for a new subnet.

SYNTHESIZING TO SHOW PRIORITIES IN A SUB-NETWORK

Open the Benefits subnet by double-clicking on the word Benefits. Select the Computations, Synthesize command in the Benefits Subnet to show the results under Benefits. You see that the European car has the greatest benefits as shown in Figure 34.

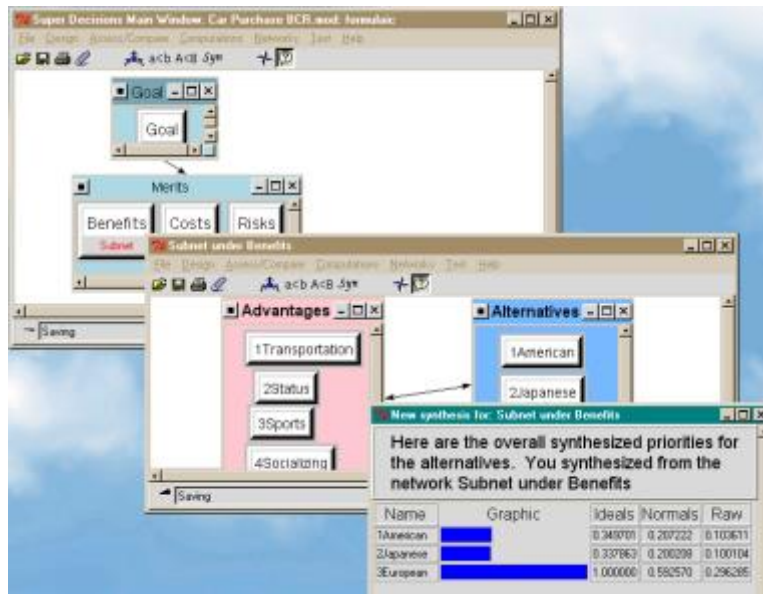
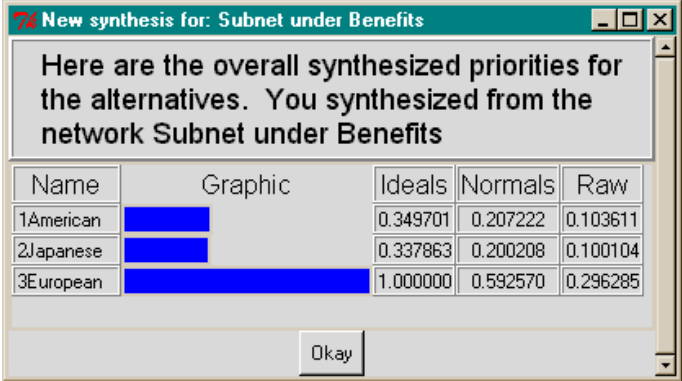
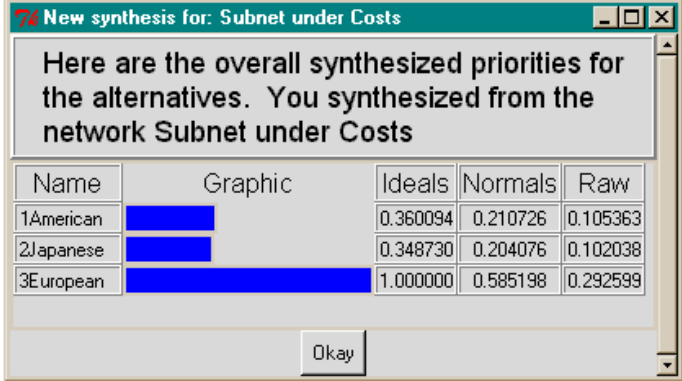
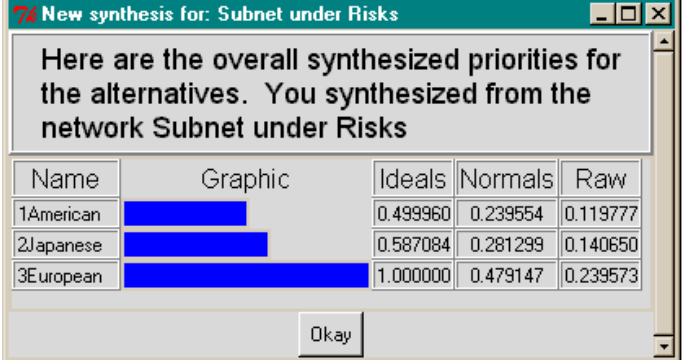


Figure 34. The Synthesis Results in the Benefits Subnet.

The synthesized priorities for each of the three subnets are shown in Table 6 below. In the Costs network and the Risks network the comparison questions are phrased this way: “Which is the most costly?” and “Which is the most risky?” Thus the European car has a 1 in the Ideals column for COSTS in Table 22 which means it is the *most* costly to buy. It also has a 1 in the Ideals column for RISKS so it is also the most risky. The sub-network results are shown in Table 22, and the results as combined in the top-level network in Table 23.

Table 22. Results in the Subnets of the Car Purchase BCR model.

BENEFITS	 <p>European is best in the Benefits subnet</p>
COSTS	 <p>European is most costly in the Costs subnet.</p>
RISKS	 <p>European is most risky in the Risks subnet.</p>

The overall results are obtained by synthesizing in the top-level network using the Computations, Synthesize command as shown in Figure 35. This combines the results from the subnets according to the formula $(\text{Benefits}/(\text{Costs} \times \text{Risks}))$ for each alternative.

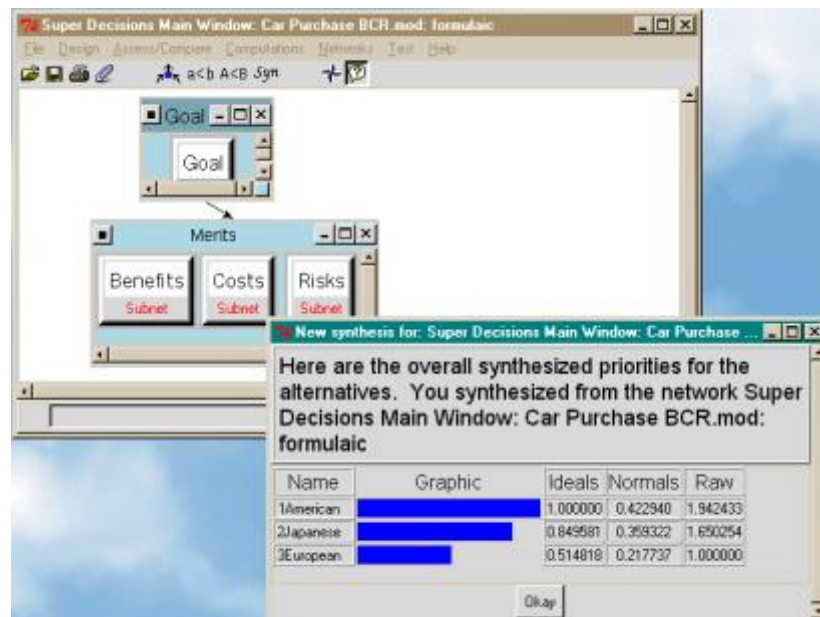


Figure 35. The *Car Purchase BCR* Overall Results.

Select the command Design, Add/Edit Formula shown in Figure 36. This opens the Formula dialogue box shown in Figure 37.

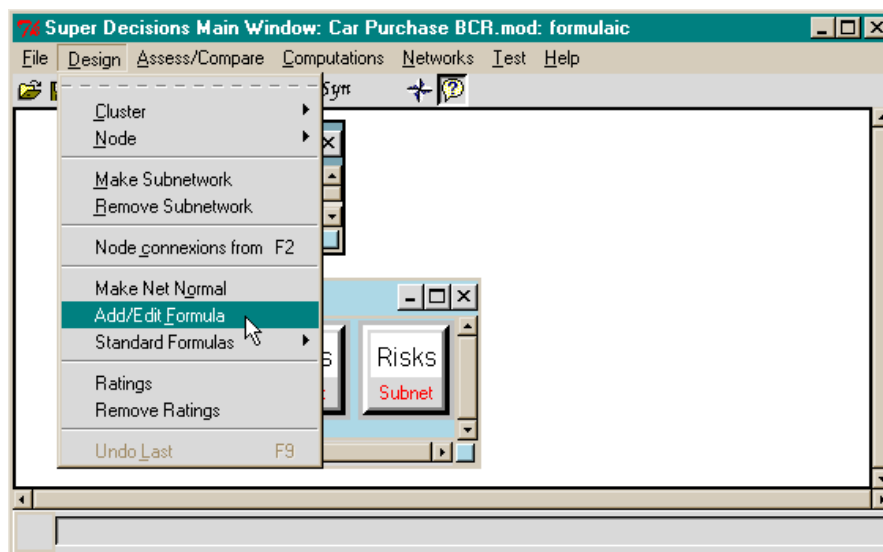


Figure 36. The Menu Command that Shows the Formula.

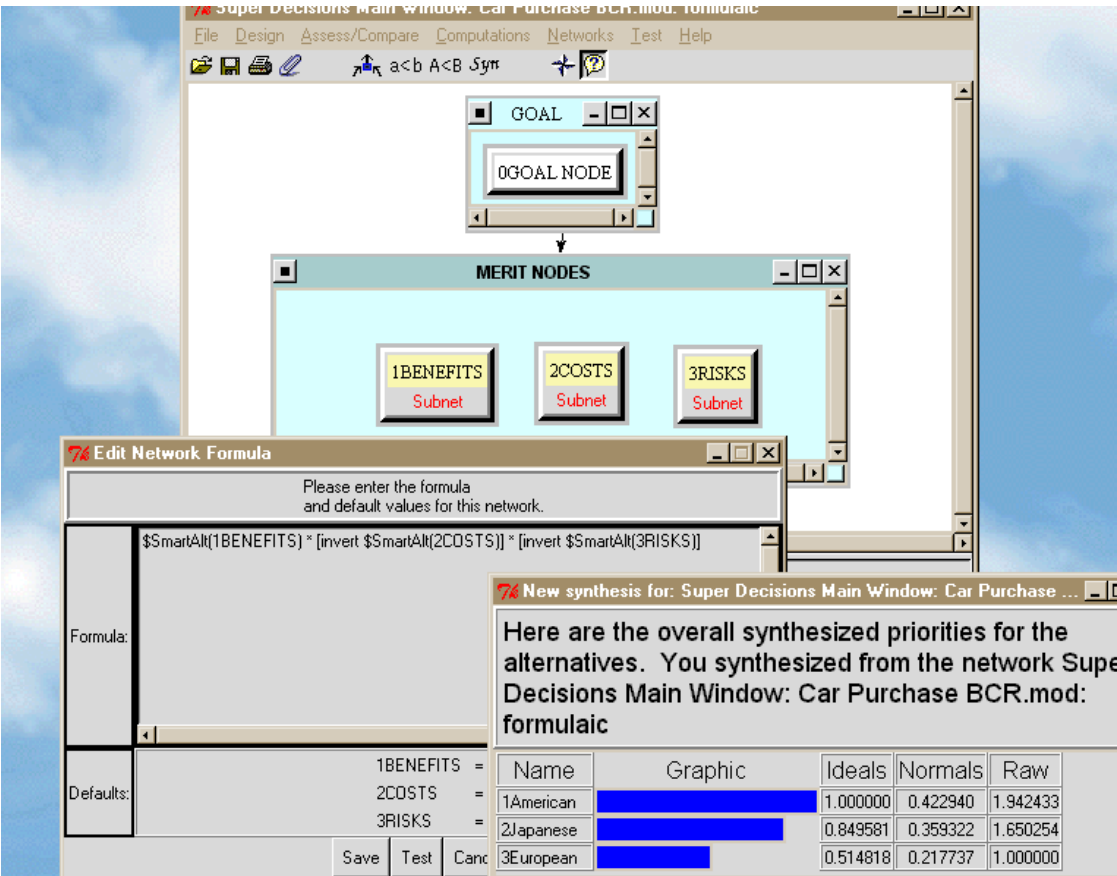


Figure 37. The Multiplicative Formula for Combining Results from Subnets in *Car Purchase BCR Model*

In the term in the formula, $\$SmartAlt(Benefits)$, the \$ prefix indicates that the term is to be calculated for each alternative and $SmartAlt(Benefits)$ means to use the appropriate values from the network beneath the Benefits node. This gives the results shown in the column under BENEFITS in Table 23.

The Multiplicative Formula is used in Figure 37. The Computations Synthesize command displays results for the alternatives in three ways: as Ideal, Normal and Raw numbers. Smart means use the Raw numbers if the network has more networks beneath it; and use the Ideal numbers if the network is either a bottom level network, or a network that has Ratings. So in this case Smart is choosing the Ideal numbers. The *invert* function in the Costs and Risks terms inverts the values; for example, Costs is inverted to $1/Costs$. For more discussion of formulas see the section on Formulas at the end of this chapter.

Now change to the additive formula by selecting the command Design, Standard Formulas, Additive. Synthesis now gives the results shown in Figure 38.

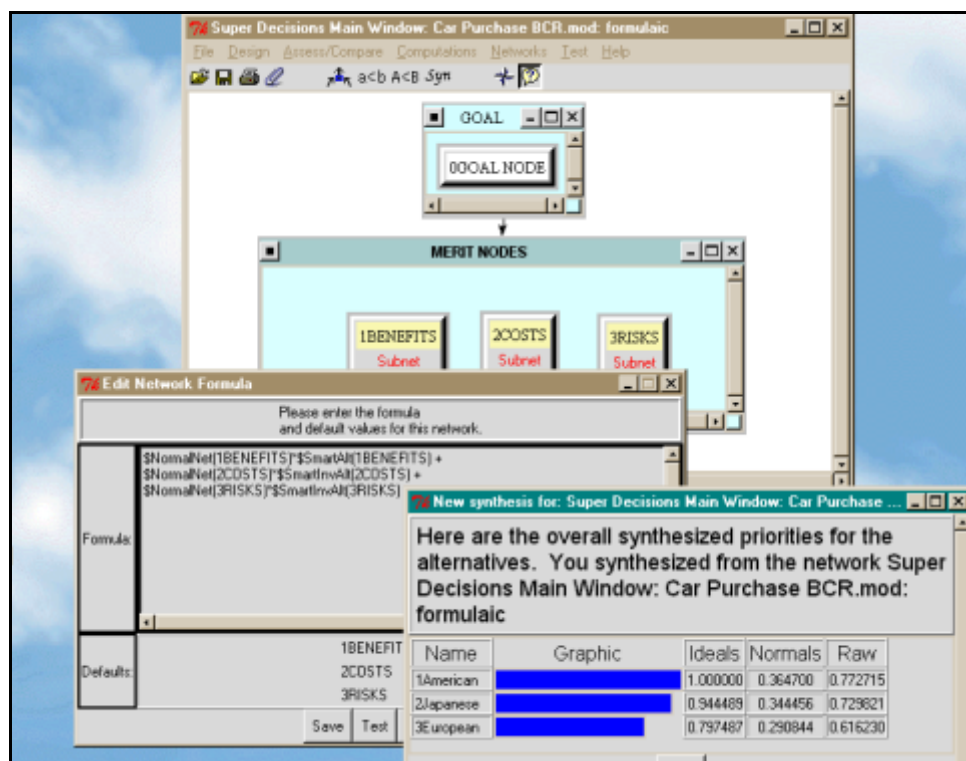


Figure 38. Synthesis Using the Additive Formula

The results are shown in two ways in Table 23: using the multiplicative formula and using the additive formula. In the additive formula the priorities in the Costs and Risks subnets have to be flipped from the most costly (risky) alternative having the highest value to the least costly (risky) alternative having the highest value. See the section on Formulas at the end of the chapter for how to do that.

Table 23. Combining Priorities in the Top-Level Network in Two Ways

	BENEFITS	COSTS	RISKS	B/(C*R) MULTIP. FORMUL A	B+1/C+1/R ADDITIVE FORMUL A
American Car	.104	.105	.120	.423	.365
Japanese Car	.100	.102	.141	.359	.344
European Car	.296	.293	.240	.218	.291

ANALYSIS OF RESULTS FOR THE CAR PURCHASE BCR MODEL

To analyze the outcome for the *Car Purchase BCR* model shown in Table 23, we can observe that the European car has the most benefits by far. But it is also the most costly, both to purchase and repair, and the most risky, because of its greater likelihood of being stolen. It has the lowest overall priority. Netting it all out the American car is best over all, mostly because of its lower risks. The ranks are the same for the cars using either formula for synthesis.

SENSITIVITY GRAPH

Now we will show how the results would change if the importance of the Benefits node were changed. Make sure the additive formula is selected (the sample model has the formula set to additive). See the sensitivity graph in Figure 39 with Benefits selected as the Independent Parameter. But the sensitivity

graph shows that at a Benefits value (on the x-axis) of about .46 the best car changes from American (the top line on the left axis) to Japanese (the second line from the top on the left axis). In the software it is easy to see which line goes with which car when viewing the screen as they are color-coded. Sensitivity can be performed only when the additive formula is selected because in the multiplicative formula the weights of B, C and R are the same for each alternative and cancel out. See the Formulas section at the end of the chapter for more about this.

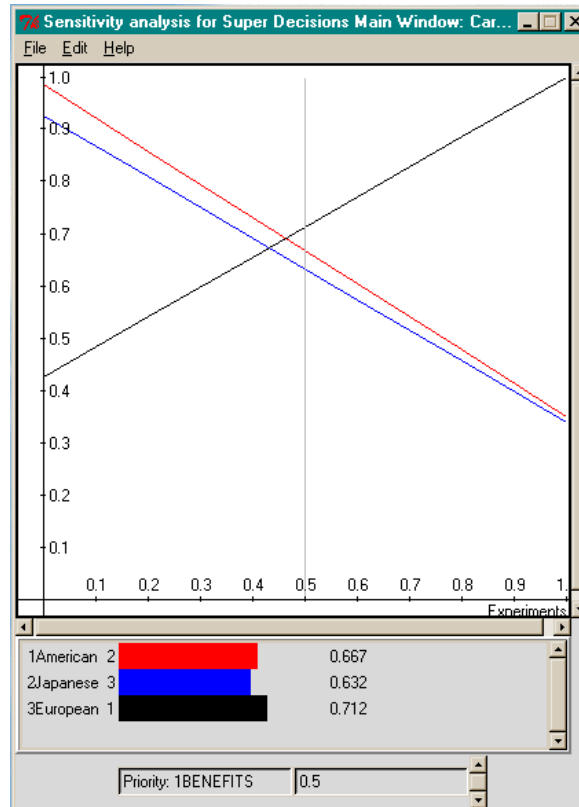


Figure 39. Car Purchase Sensitivity Graph for Benefits (Additive Formula)

DEMONSTRATION OF A COMPLEX THREE-LAYER SYSTEM: THE NATIONAL MISSILE DEFENSE MODEL (NMD)

This application is available for examination in the sample models of the *SuperDecisions* software. The file is called *NationalMissileDefense.sdmod*. This model was presented by Thomas L. Saaty at the 6th International Symposium on the AHP in Bern, Switzerland, 2001. This model is a complete multi-layer structure with Benefits, Opportunities, Costs and Risks (BOCR) merit nodes in the top-level network, control criteria in their attached subnets, and bottom level decision subnets containing the alternatives that are attached to the control criteria. The top-level model also has an attached Ratings component for evaluating the importance of the BOCR. An overview of such a complete model structure is shown in Figure 40.

The United States government faces the crucial decision of whether or not to commit itself to the deployment of a National Missile Defense (NMD) system. Many experts in politics, the military, and academia have expressed different views regarding this decision. The most important rationale behind supporters of the NMD system is protecting the US from potential threats said to come from countries

such as North Korea, Iran and Iraq. According to the Central Intelligence Agency, North Korea's Taepo Dong long-range missile tests were successful, and it has been developing a second generation capable of reaching the US. Iran also tested its medium-range missile Shahab-3 in July 2000. Opponents express doubts about the technical feasibility, high costs (estimated at \$60 billion), political damage, possible arms race, and the exacerbation of foreign relations.

The current plan for NMD originated with President Reagan's Strategic Defense Initiative (SDI) in the 1980s. SDI investigated technologies for destroying incoming missiles. The controversies surrounding the project were intensified with the National Missile Defense Act of 1996, introduced by Senator Sam Nunn (D-GA) in June 25, 1996. The bill required Congress to make a decision on whether the US should deploy the NMD system by 2000. The bill also targeted the end of 2003 as the time for the US to be capable of deploying NMD.

The next year the Senate Armed Services Committee approved the National Missile Defense Act of 1997 by winning 10 votes out of 18, along party lines. This Act mandated deployment of an anti-missile system, consisting of 100 ground-based interceptor missiles at a single site, plus ground-based radars and space-based sensors. The intelligence community estimated a shortened warning time for the US against intercontinental ballistic missiles (ICBMs) deployment. However, the deployment of NMD by 2003, analyzed by an independent Commission to Assess the Ballistic Missile Threat to the United States, concluded that it would generate high risks and possible failure. Accordingly, the Administration adjusted its plan to deploy an NMD in 2005.

SCHEMATIC OF A COMPLEX MODEL

This is the type of model generated when you select the File, New command and Full Model template.

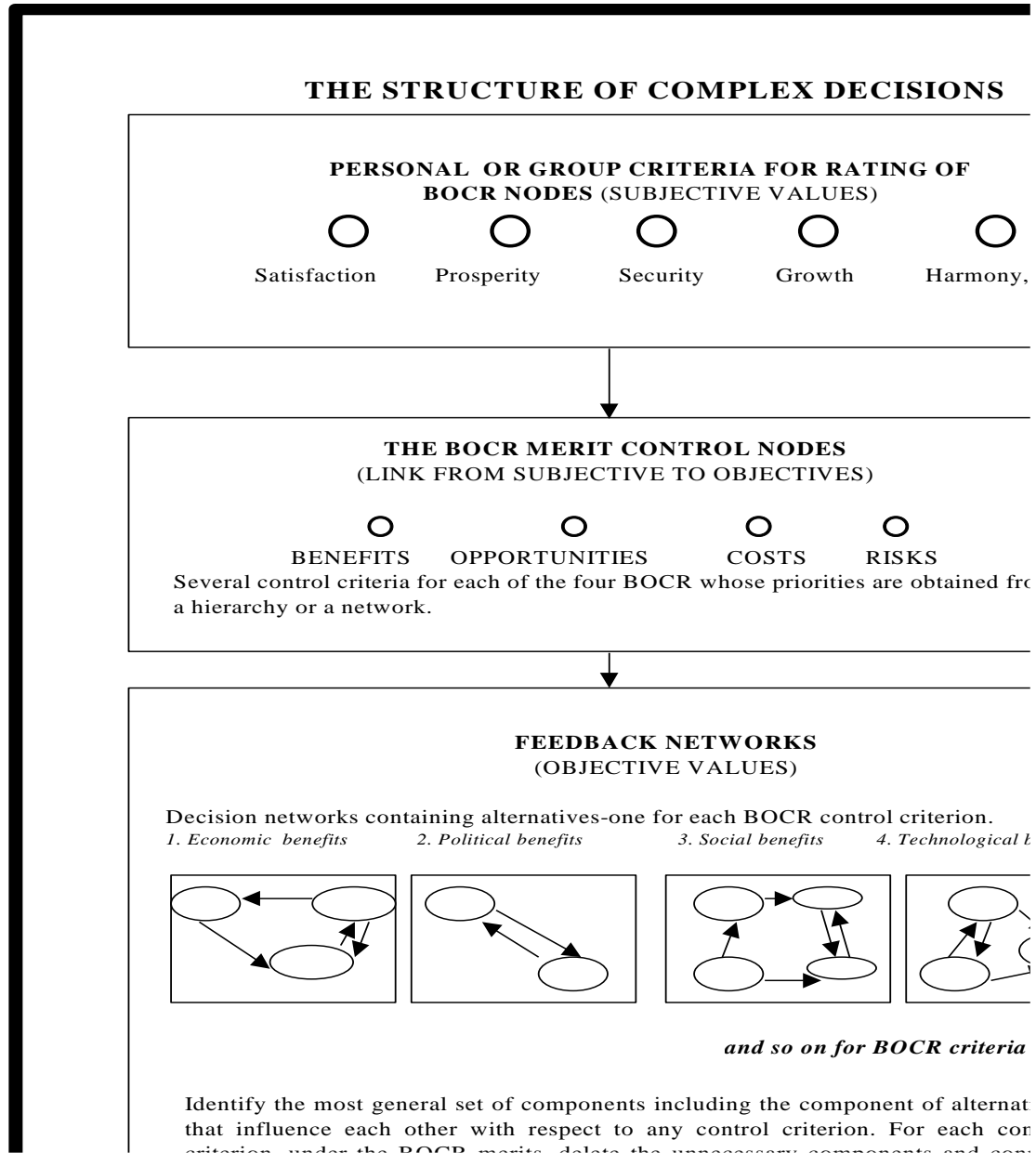


Figure 40. The Structure of a Full Model for Complex Decisions

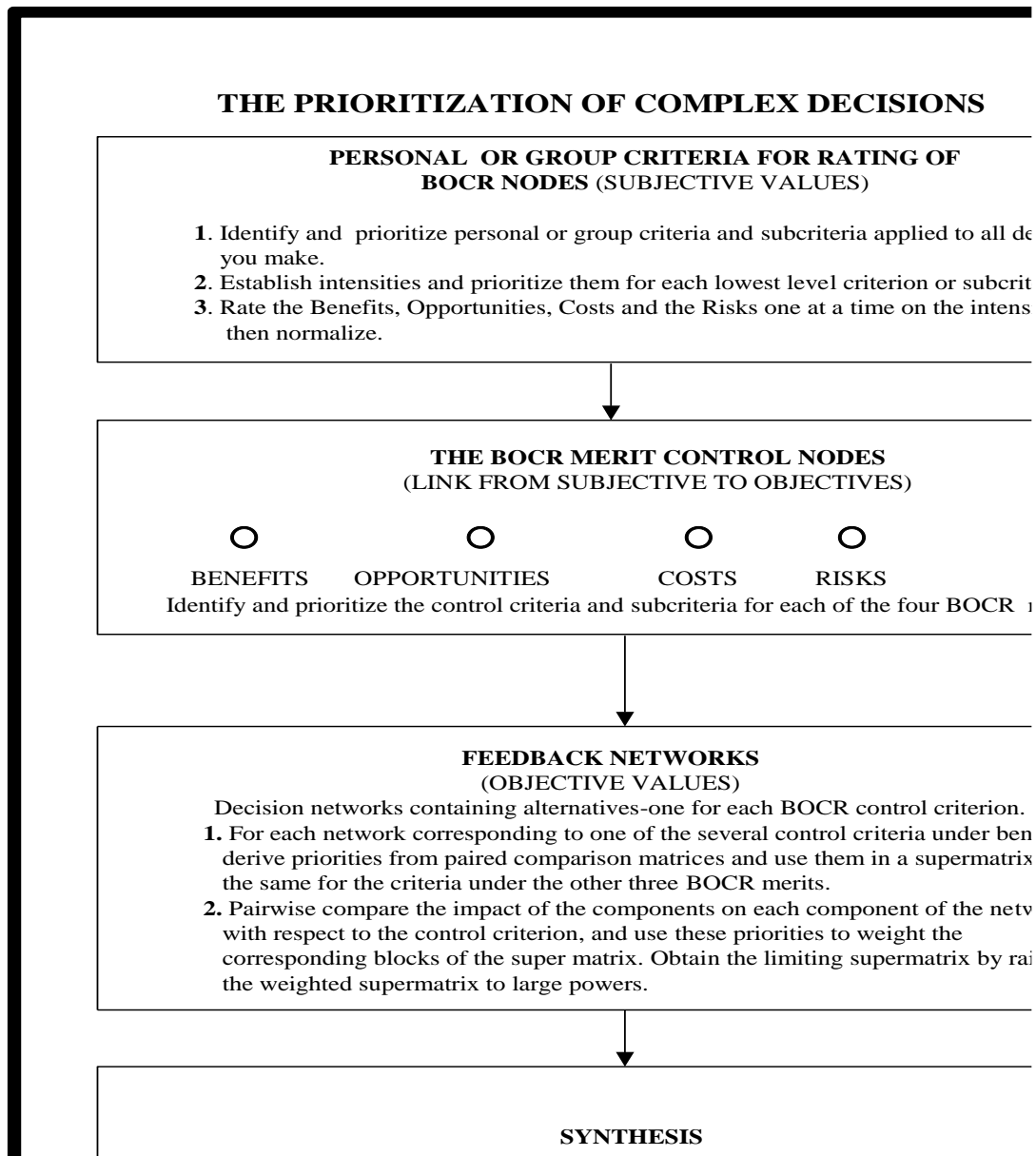


Figure 41. The Prioritization of Decisions

The deployment of NMD is not solely based on technological development. The next president of the US has to deal with international politics. The Anti-Ballistic Missile (ABM) treaty signed by the US and the former Soviet Union in 1972 would ban NMD, and the next president should be able to persuade or renegotiate the ABM treaty with Russia's president, Vladimir Putin, who strongly opposes the plan. How to deal with the reactions of China and NATO is another issue for the president after Mr. Clinton to consider. This analysis was done in October 2000 when Mr. Clinton was still President. Since then, George W. Bush has become President, and his Secretary of State, Colin Powell, has made statements supporting the NMD deployment.

OVERVIEW OF THE NMD MODEL

Under the situation in October 2000, what is the best direction for NMD to take? The following alternatives and criteria for evaluating the decision were identified.

1. **Deploy NMD.** Fully deploying the NMD program
2. **Global Defense.** Amending the ABM treaty to be more restrictive to more countries by using any economic, political, and diplomatic means as well as implementing joint-development of a worldwide defense system.
3. **R & D.** This alternative is not concerned with deployment, but proceeds with research and development of missile defense technology.
4. **Termination of the NMD program.** Disregarding any further R&D and deployment plan.

The evaluation criteria are categorized into benefits, costs, opportunities, and risks. A three-layer network model was developed, using the *SuperDecisions* software:

- **Top-level Network** This is a single network that has in it the benefits, opportunities, costs, and risks nodes (the BOCR nodes) and the strategic criteria used to evaluate their importance for this decision. This network has an associated Ratings spreadsheet that is used to evaluate the BOCR under the strategic criteria.
- **Control Criteria Networks** Each of the BOCR has a subnet attached to it containing its control criteria. Usually the structure in these subnets is hierarchical. The most important control criteria are selected to have decision subnets created for them.
- **Decision Networks.** A decision subnet is created for each high value control criterion. The alternatives of the decision appear in a cluster in each decision subnet, but other than that the clusters and nodes may be different.

THE TOP-LEVEL NETWORK

The assessment criteria used to determine the priorities of the BOCR merits are shown in Figure 42. The decision on the NMD project is reviewed in the context of the three criteria that are used to evaluate the merits. These are World Peace, Human Well-being, and International Politics. The three sub-criteria, Adversary Countries, Security Dilemma, and Terrorism cover all the causes disturbing or stabilizing peace in the world. The first sub-criterion, Adversary Countries concerns the potential threats by adversary countries. The second criterion, Security Dilemma means that increasing one country's security inevitably decreases other countries' security. Terrorism indicates any possibility of the rise or decline of terrorism in the world. Human Well-being includes Technological Advancement and Market Creation. Technological Advancement driven by the NMD research and development process can ultimately benefit all people, particularly in providing possible space exploration that can lead to the creation of new markets. Moreover, the 21st century is characterized as a post-industrialization era. Service industries in communication and transportation will benefit not only businesses associated with these industries, but also consumers who can enjoy the products from the new market. The last criterion is International Politics. It is composed of two sub-criteria, Military Relations and Diplomatic Relations.

Military Relations refer to the impact of NMD on relations with US allies for better or for worse. Also, the impact of NMD on diplomatic relations among all countries should be considered.

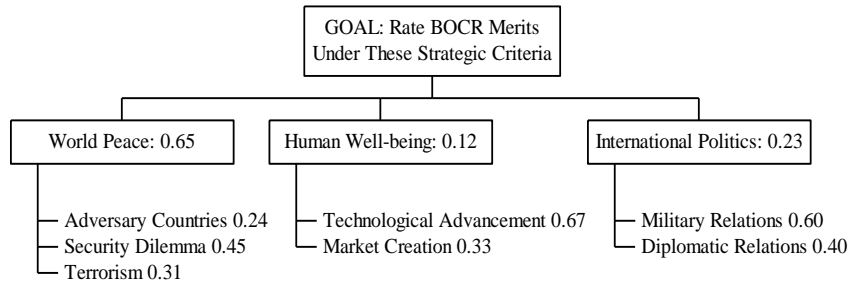


Figure 42. Strategic Criteria for Rating Benefits, Opportunities, Costs and Risks (BOCR)

We will now turn to the view in the *SuperDecisions* software. The strategic criteria and the BOCR nodes are in the top-level network shown in Figure 43. The nodes in the Criteria cluster are pairwise compared with respect to the goal in the Strategic Criteria. The file name for this model is *National Missile Defense.sdm* and it is included in the sample models. The file name is shown on the title bar of the top-level network window. The title bar also includes the word *formulaic*, indicating that a formula is associated with this network for combining the results from the subnets underneath the BOCR nodes. And, finally, the word *ratings* appears on the title bar to show that a ratings spreadsheet is associated with this network.

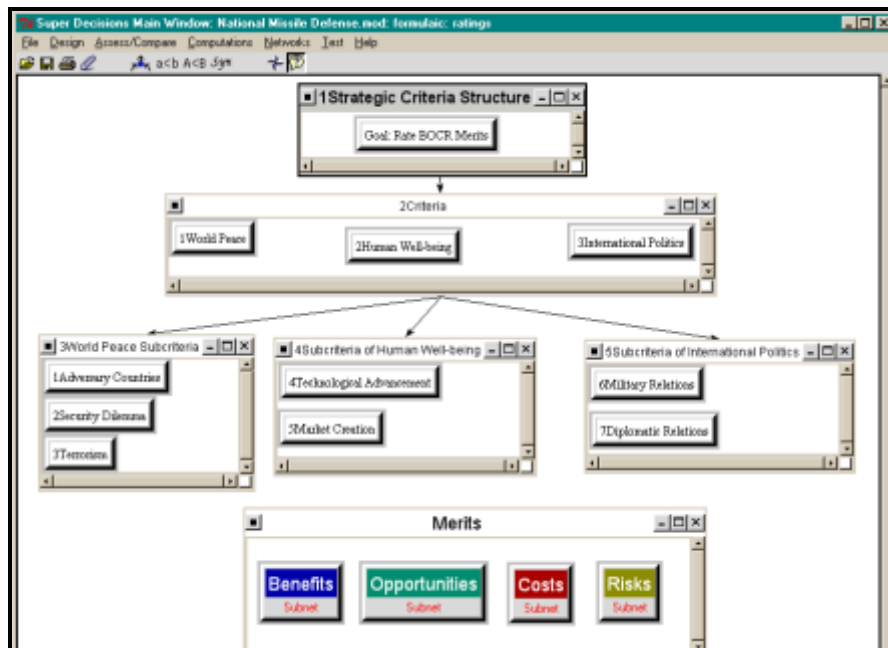


Figure 43. Top-level Network of the *National Missile Defense* Model.

The ratings spreadsheet for rating the BOCR is shown in Figure 44 along with the priorities derived for them. To access it use the command Assess/Compare, Ratings in the network to which it is attached. To create it in the first place, use the command Design, Ratings. This will also access it if it was previously created. The values for the subcriteria nodes in Ratings can be read in the main network from the Computations, Limit Matrix in the column headed Goal.

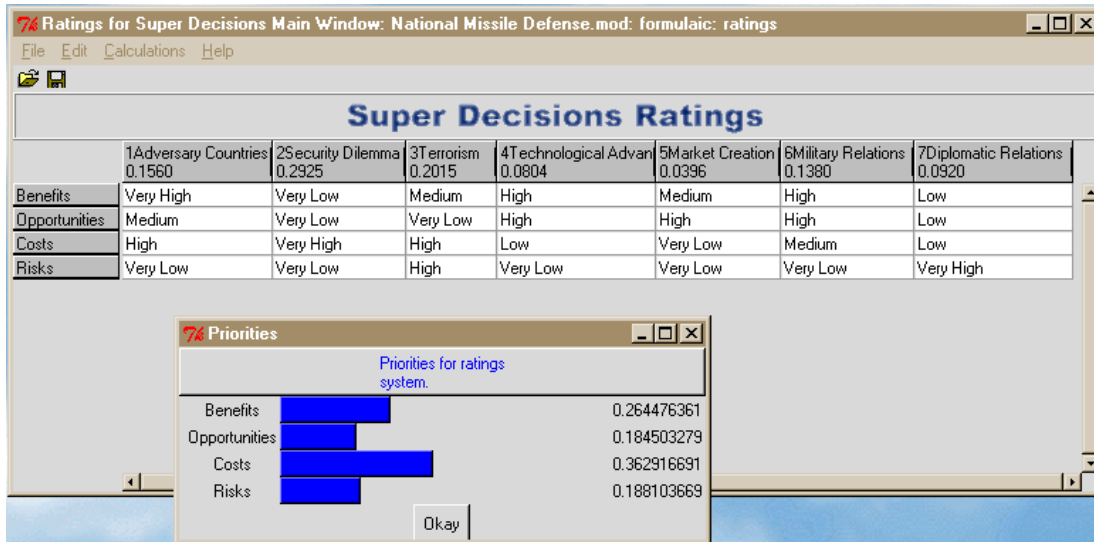


Figure 44. Ratings Spreadsheet and Resulting BOCR Priorities for Top-Level of NMD Model.

To complete the story, the synthesis command uses the formula for the top-level network shown in Figure 45. Do not be alarmed by its seeming complexity. It simply specifies that the values for the BOCR as determined in Ratings are used to weight the values for the alternatives coming up from the subnets under the BOCR and the four BOCR terms are then added and normalized to obtain the synthesized results for each alternative. Formulas are discussed in more detail at the end of this chapter.

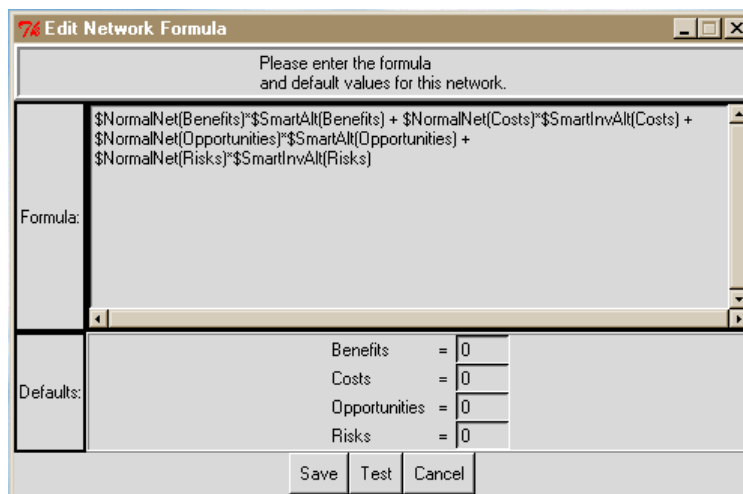


Figure 45. The Formula Associated with the NMD Top-level Network.

THE CONTROL CRITERIA NETWORKS

Each of the BOCR has an attached control criteria network. A partial view of the Benefits subnet is shown in Figure 46.

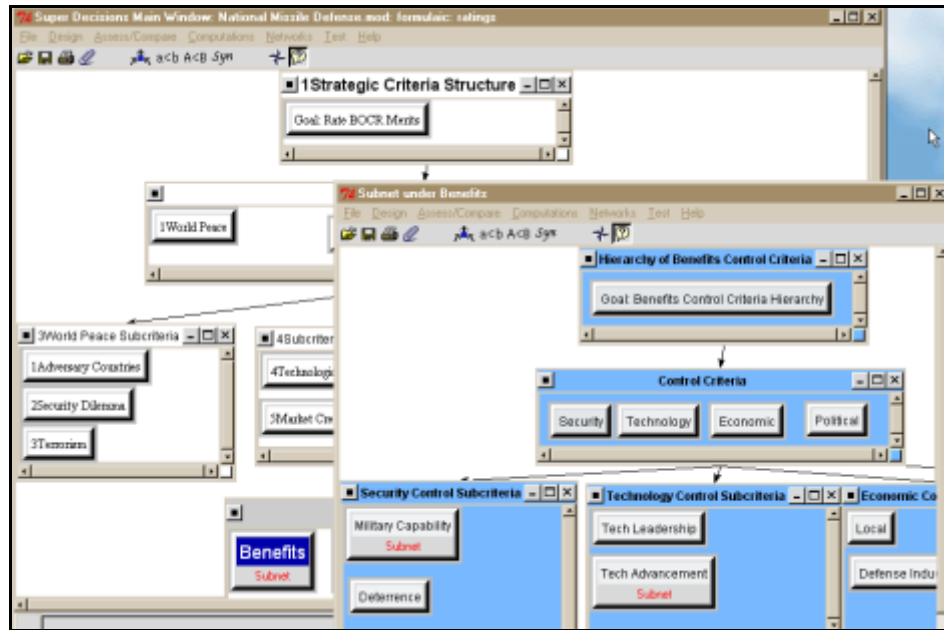


Figure 46. The Control Criteria Subnet belonging to Benefits

In this Benefits subnet the four main criteria of Security, Technology, Economic and Political are pairwise compared with respect to the Goal. Then the subcriteria of Security, Military Capability and Deterrence are pairwise compared with respect to Security, and so on. The most important potential control criteria are Military Capability (.28) and Tech Advancement (.24), together having .52 or more than half of the priority. These values can be read from the Computations, Limit Matrix in the column headed Goal.

THE DECISION NETWORKS

We will examine one bottom level decision network, the one attached to the Military Capability control criterion, shown in Figure 47 in detail. The decision makers: Congress, and the executive branch consisting of the President and the Department of Defense (the node name is military). Technical experts and the Defense Industry influence Congress and the executive branch by providing their professional expertise and technical information.

Congress, Military, Defense Industry, and Technical Experts all have a say about the extent to which the alternatives contribute to the Military Capability of the US. The results are that Deploy NMD will increase the military capability followed by Global Defense, R&D and Termination but to very different degrees. Table 3 shows the 23 control criteria under the benefits, opportunities costs and risks and their priorities.

Table 3 Criteria and Their Priorities

Merits	Criteria	Subcriteria	Local Priorities	Global Priorities
Benefits (0.264)	Economic (0.157)	Local Economy	0.141	0.006
		Defense Industry	0.859	0.036
	Political (0.074)	Bargaining Power	0.859	0.017
		U.S. Military Leadership	0.141	0.003
	Security (0.481)	Deterrence	0.267	0.034
		Military Capability	0.590	0.076
		Anti-terrorism	0.143	0.018
	Technology (0.288)	Tech. Advancement	0.834	0.064
		Tech. Leadership	0.166	0.013
Opportunities (0.185)	Arms Sales		0.520	0.094
	Spin-off		0.326	0.059
	Space Development		0.051	0.009
	Protection of Allies		0.103	0.019
Costs (0.363)	Security (0.687)	Security Threat (Vulnerability to the security threat)	1.000	0.248
	Economic (0.228)	Sunk Cost	0.539	0.044
		Further Investment	0.461	0.038
	Political (0.085)	ABM Treaty	0.589	0.018
		Foreign Relations	0.411	0.013
Risks (0.188)	Technical Failure		0.430	0.082
	Arms Race		0.268	0.051
	Increased Terrorism		0.052	0.010
	Environmental Damage		0.080	0.015
	U.S. Reputation		0.170	0.032

The Merits values in the first column are obtained by using the Calculations, Priorities command in Ratings. The criteria and subcriteria values are obtained by going into the control criteria subnet (for Benefits for example), using the Computations, Unweighted Supermatrix command and reading the values from it. The Global Priorities values are obtained using Excel. For example, the first one, 0.006, is obtained by multiplying $0.264 \times 0.157 \times 0.141$.

The 23 criteria are in the control criteria subnets attached to the BOCR. They were prioritized through pairwise comparisons. Among these 23 criteria, the sum of the priorities of 9 of them, security threat, arms sales, technical failure, military capability, technological advancement, sunk cost, spin off, arms race, and further investment account for over 0.76 of the total. To economize effort, we used 9 to do the analysis. We renormalize their priorities within their respective merits and continue. One of these subnetworks is shown in Figure 47.

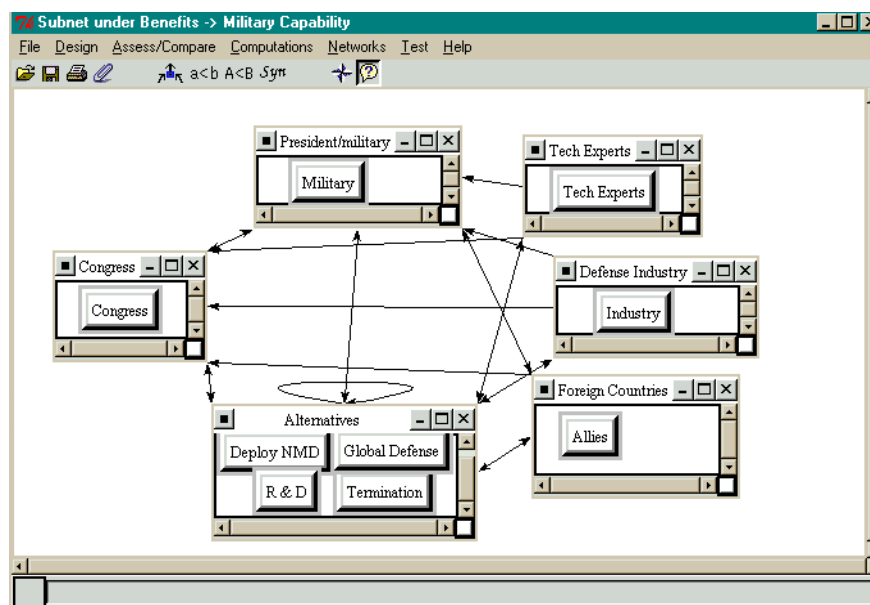


Figure 47. Military Capability Sub-network Under Benefits

Table 5 shows the final synthesis of the alternatives for each of the BOCR merits and the overall result, using the reciprocals of the synthesized priorities of costs and risks.

Table 5 Final Outcome

	Benefit s (0.264)	Opportuniti es (0.184)	invCost s (0.363)	invRisks (0.188)	Final Outcome using Additive (norm.)	Final Outcome using Multiplicative (norm.)
Deploy NMD	0.434	0.473	0.306	0.116	0.331	0.493
Global Defense	0.357	0.290	0.305	0.178	0.291	0.379
R & D	0.161	0.151	0.236	0.289	0.212	0.110
Terminatio n	0.049	0.085	0.153	0.417	0.165	0.018

The final outcome is calculated in two ways: using an additive formula and a multiplicative formula. The effect of using the multiplicative formula is that the weights of the BOCR cancel out due to the form of the formula, so in effect they are equal at 0.25. The reason the Deploy option is so much better under the multiplicative formula is that costs are very high for that alternative, thus when costs are weighted by .363 as they are in the additive formula, it drags down the value of the Deploy option.

OVERALL OUTCOME AND SENSITIVITY ANALYSES

Let us look at sensitivity for results using the additive formula. Deploy NMD at 0.331 is the best option. It is a comprehensive result that takes into consideration all benefits, opportunities, costs, and risks. First, we let us examine sensitivity if the value of Benefits changes. No matter what the value of Benefits, the Deploy option (the top line) is dominant. There is one line for each alternative in sensitivity windows. In the software they are color coded so that it is easy to see which line corresponds to each alternative, but here in black and white they look the same.

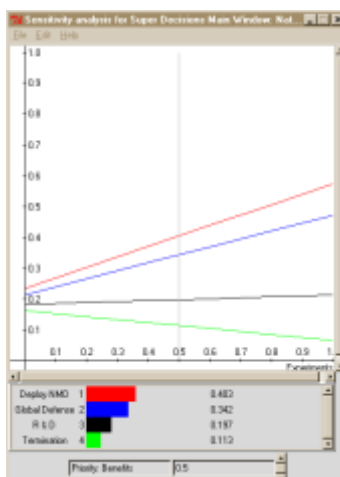


Figure 48. Sensitivity Analysis: Benefits

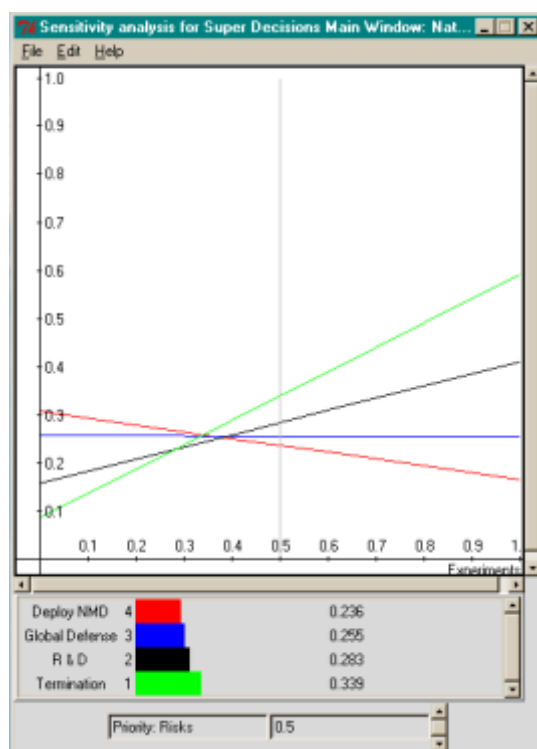


Figure 49. Sensitivity Analysis: Risks

We did similar sensitivity tests for some of the control criteria in the subnets as shown below. We found that the outcome was very stable and did not change the overall ranks except for changes of the 3 criteria, security threat, further investment, and sunk cost, all under costs. When the priority of the security threat decreases to 0.154 from 0.248 or the priority of the further investment increases to 0.774 or the priority of the sunk cost increases to 0.803, then the rank of termination changes from third to second to first.

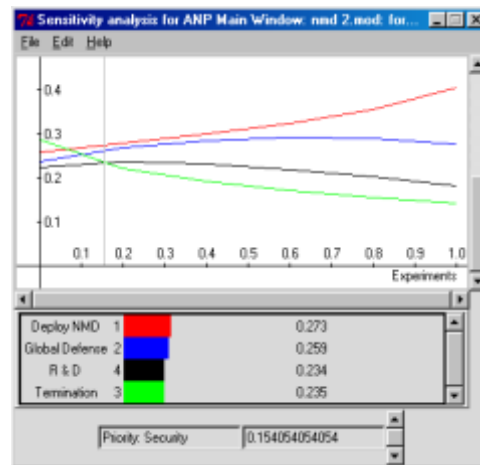


Figure 50. Sensitivity Analysis: Security Threat

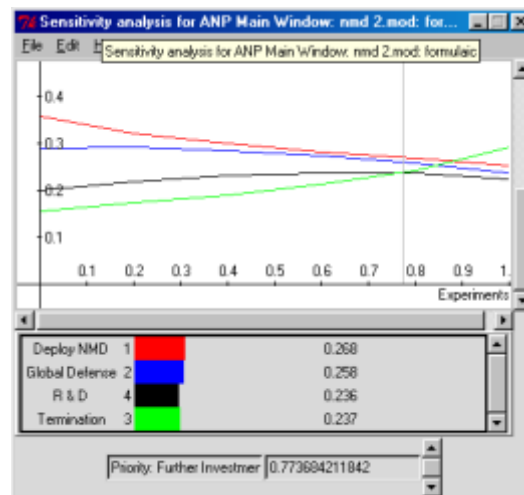


Figure 51. Sensitivity Analysis: Further Investment

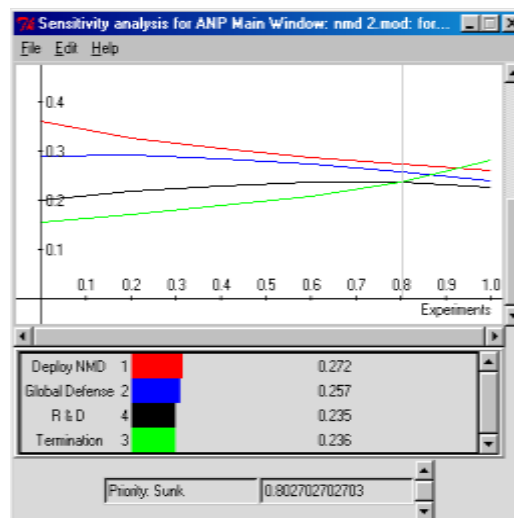


Figure 52. Sensitivity Analysis: Sunk Cost

OVERVIEW OF FORMULAS FOR MULTI-LAYER MODELS

In general formulas are used only in the top-level network of a multi-layer system for synthesizing the results as they feed up from the subnets. No formulas are used in the bottom level subnets as they simply feed the synthesized values for the alternatives upward, and they continue to rise up through any number of intervening sub-networks until they reach the top. See the section on Formulas at the end of the chapter for more information on how to select and input a formula. The Additive formula was used in the NMD model.

HOW TO BUILD AND GET RESULTS IN A MODEL WITH SUBNETS- A WALKTHROUGH

Here we will show you step by step how to build the car model that was demonstrated earlier in the chapter. We shall name the new one the *aejcar.sdm* sample model. The end result will be the same as the *Car Purchase BCR* model if you use our inputs, but use your own if you wish.

The model you will build contains a control network and three sub-networks associated with the Benefits, Costs and Risks control criteria in the control network. Networks are built of clusters that contain nodes with links between the clusters and links between the nodes. The nodes in a cluster have something in common, without being very specific about what it is, that makes them related in some sense. This model is to help a car buyer prioritize, in terms of overall satisfaction, the type of car he or she prefers, American, Japanese or European. The types of cars or carmakers (American, Japanese, or European) are the alternatives. One requirement for an ANP model is that there must be a cluster labeled “Alternatives” in each sub-network that contains the alternatives of the problem as nodes. Start the model by entering the three control models, their clusters and the elements in the clusters as shown in Table 24 .

Table 24. The Nodes You Will Create in the *aejcar* Model.

Control Criterion	Clusters in Sub-Model	Nodes in Clusters
Benefits criterion	Advantages	Transportation, Status, Sports, Socializing
	Alternatives	American, Japanese European
Costs criterion	Outlays	Initial Cost, Repair Cost, Reliability Cost
	Alternatives	American, Japanese European
Risks criterion	Bad Luck Events	Accidents, Stolen, Repossession
	Alternatives	American, Japanese European

CREATING A MODEL

Load the ANP software. The opening screen is shown in Figure 53 below. You may build a simple network in this window that comprises the whole model, or you may create some clusters with nodes in them and create sub-networks for some of the nodes. You then create clusters and nodes in the sub-networks and for some of the nodes there create sub-sub-networks. In theory there is no limit to the number of layers. The top-level model is the first layer, the sub-networks are the second layer and the sub-sub-networks are the third layer.

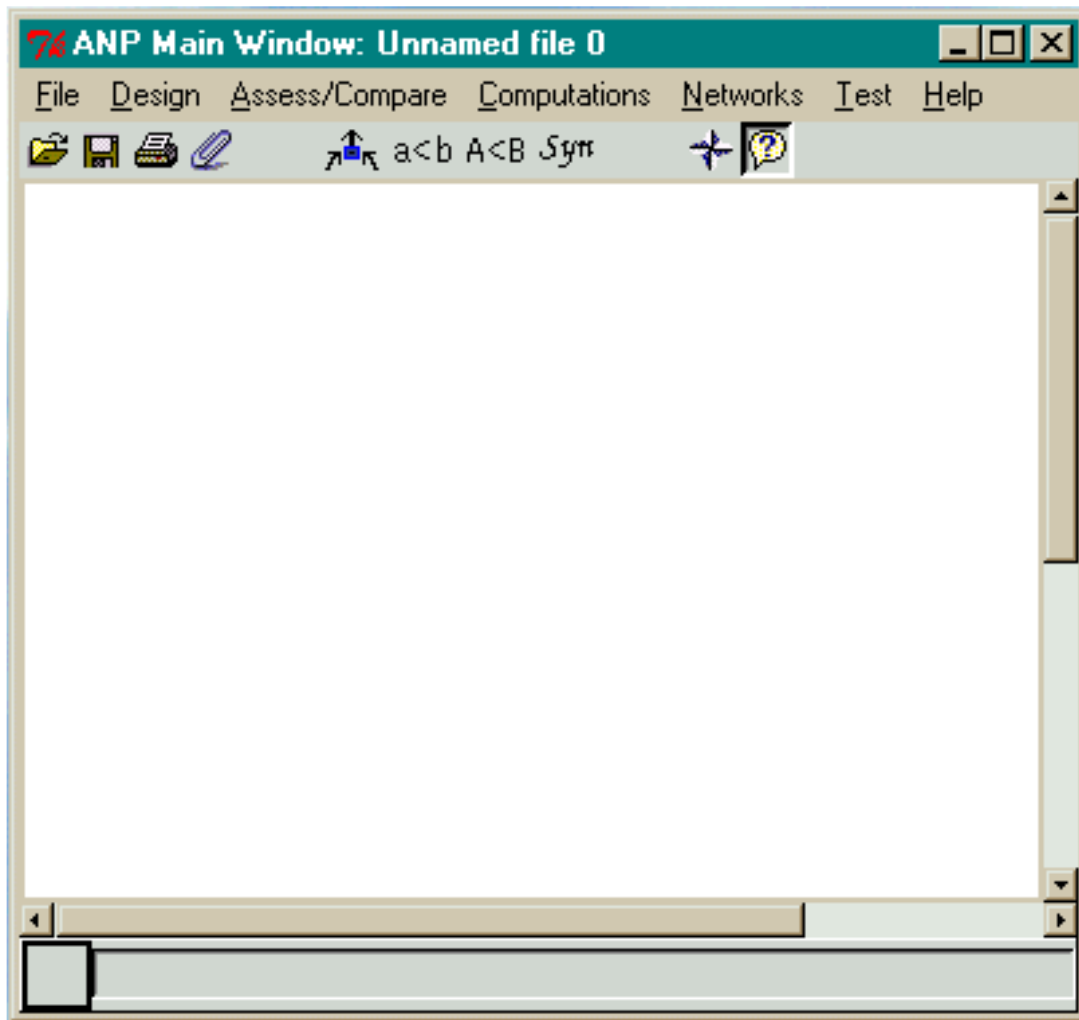


Figure 53. The Starting ANP Main or Top-level Window.

STARTING A NEW MODEL

In this section we will walk you through creating the *Car Purchase* model demonstrated earlier. Name the new model *aejcar.sdm* and create it using the Small Template. To start a new model using a template select the File New command as shown in Figure 54 below.

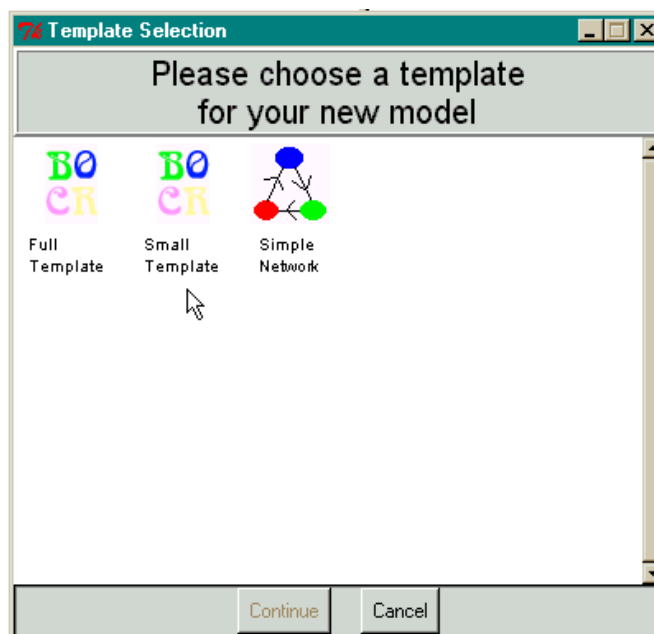
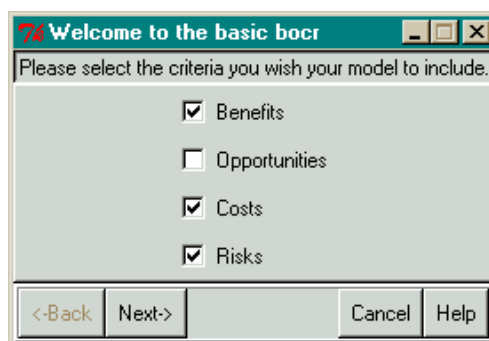


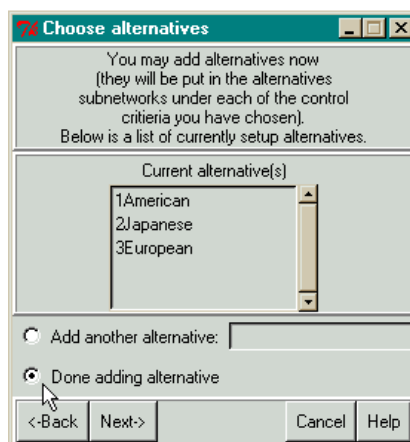
Figure 54. Select File New, then Small Template to Start the Car Model.

STARTING A MODEL USING THE SMALL TEMPLATE

1. Select Benefits, Costs and Risks



2. Add Alternatives – Select Done and Next when Finished



The automatically generated control network for a BCR model will appear in the ANP main window as shown in Figure 55 below. It has what amounts to a hierarchy with a goal in the goal cluster and the BCR merits in the cluster named Model. You could have used the *Design Cluster* and *Design Nodes* commands and built the same model yourself.

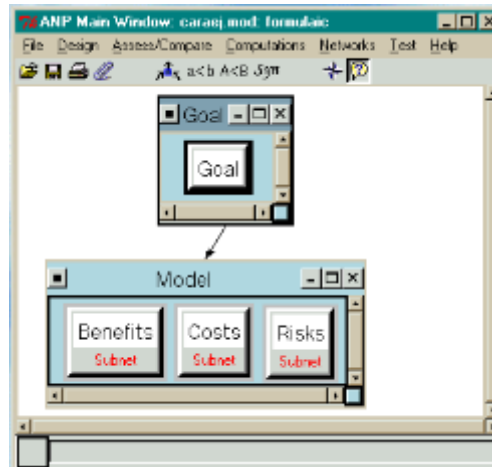


Figure 55. The Top Level Network that was Automatically Generated for the *aejcar* Model.

The Benefits, Costs and Risks sub-networks are started for you with the alternative cluster already in place. See Figure 56 below. You need to add the other clusters.

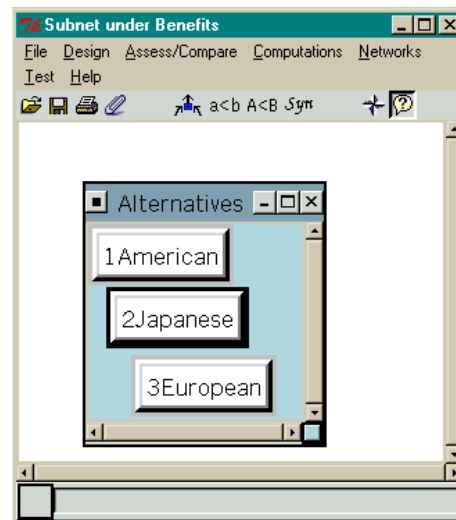


Figure 56. The Subnets Created by the Template under Benefits, Costs and Risks are the Same.

To reach a subnet, left double-click on its control node in the network immediately above the subnet, or you can use the node drop-down menu and select the *Make/show Subnetwork* command.

The clusters look like windows with scroll bars. To change the size of a cluster window click on the blue button at the lower left corner of the window and drag. To drop down a cluster menu left-click on the left top corner of the cluster as shown in Figure 57. To minimize it by changing it into an icon, click on the minus sign at the top right, or the X. To change it back from an icon, left double-click on the icon. To enlarge the cluster left-click on the “maximize” square at the top right.

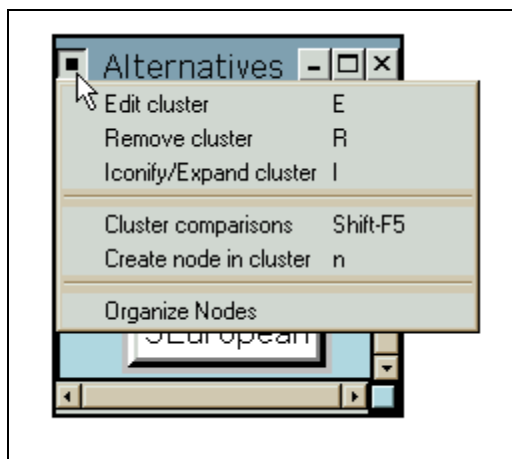


Figure 57. View of Cluster Window Icons and Menu

The Editing Cluster dialogue box is shown in Figure 58 below. You can change many things about the appearance of a cluster using the properties here.

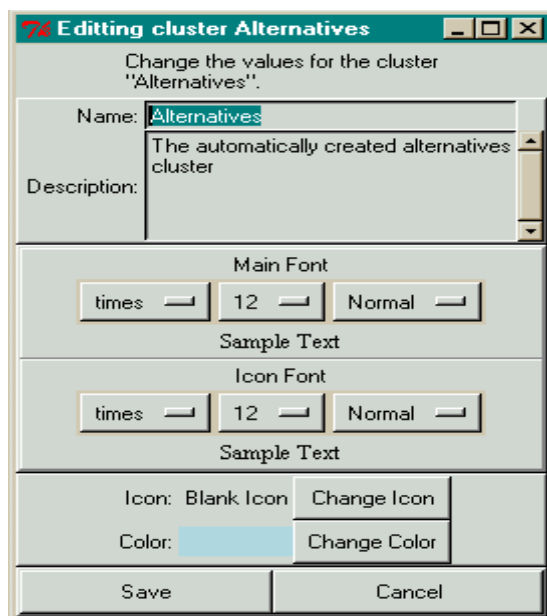


Figure 58. The Editing Cluster Dialog Box

Tip: If you want to choose a particular default font, style, and background color for clusters and for nodes before you start a new model, select the File Configure command and set them using the dialog box shown in Figure 59 before creating the control network.

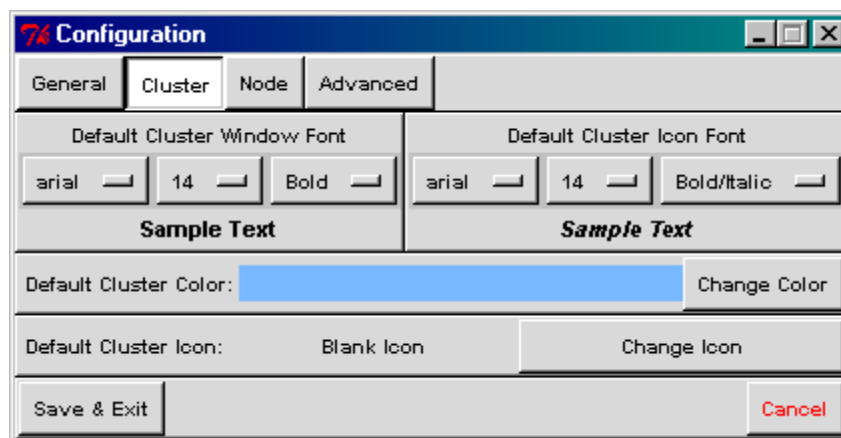


Figure 59. The Configuration Box for Customizing Cluster and Node Fonts.

FORMULAS

Using a template will assure that the formula for combining the results in the top-level network as they feed up from the subnets is correct. The only formula in a two-level model is in the top network. No formula is used in a bottom level subnet as it simply feeds the synthesized values for the alternatives upward, and they continue to pour upwards through any number of intervening sub-networks until they reach the top. The formula used in the top-level network is automatically generated when you use the Small Template wizard to start your model. Select the *Design, Add/Edit Formula* command to see the network formula that was generated, shown in Figure 60 below.

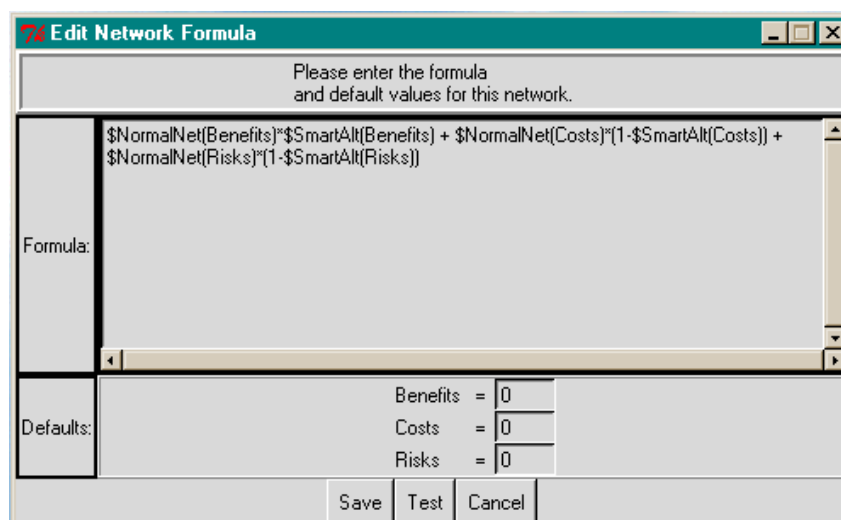


Figure 60. Subtractive Formula Produced by Template for Two-level Model.

MULTIPLICATIVE FORMULA

The standard formula that is automatically generated automatically uses the names of the control nodes that have been chosen in the template: in this case they are Benefits, Costs and Risks. It is the standard subtractive formula. See the section on formulas at the end of this chapter for more information. The formula can be changed by the user simply typing in a new one, or by selecting a different standard formula. Change the formula to the multiplicative formula using the following commands.

1. Select the *Design, Add/Edit Formula* command to see the automatically generated formula.

2. Close that and select the *Design, Standard Formulas, Multiplicative* to change the formula to the one shown in Figure 61.
3. Check the new formula by selecting the *Design, Add/Edit Formula* command again.

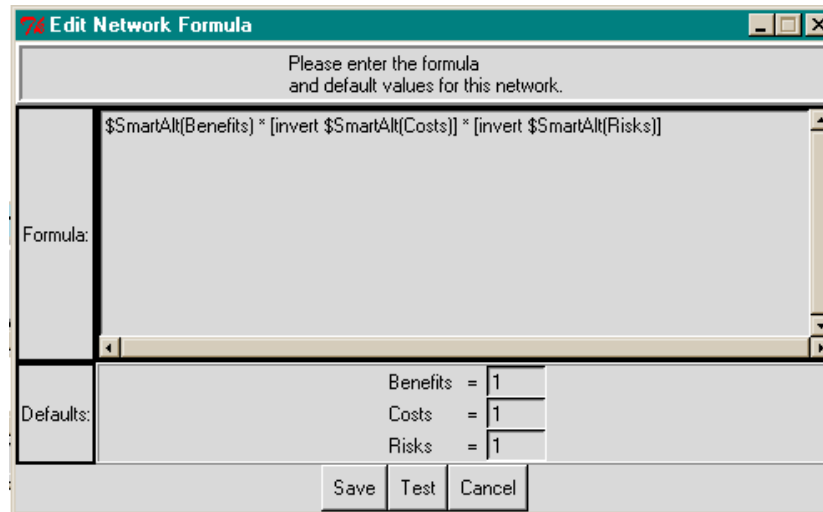



Figure 61. The Multiplicative Formula

Multiplying by the inverse of Risks and Costs ($B * (1/\text{Risks}) * (1/\text{Costs})$) is the same as dividing Benefits by Risks and Costs. The questions in the Costs and Risks subnets must be phrased by asking which is the more costly or more risky, so that the alternatives with the most cost and most risk end up with the highest priorities. The higher the Risk and Cost numbers for an alternative, the lower the B/CR result for that alternative, which is what you would want.

LINKING NODES

The simplest way to link a node is to turn on the make connections node by clicking on the  icon to depress it. Then left-click on the “parent” node and right-click on all the descendants of it (which may be in different clusters – you can connect them all in one process). You can also use the Node connections from command on the dropdown node menu shown in Figure 62.

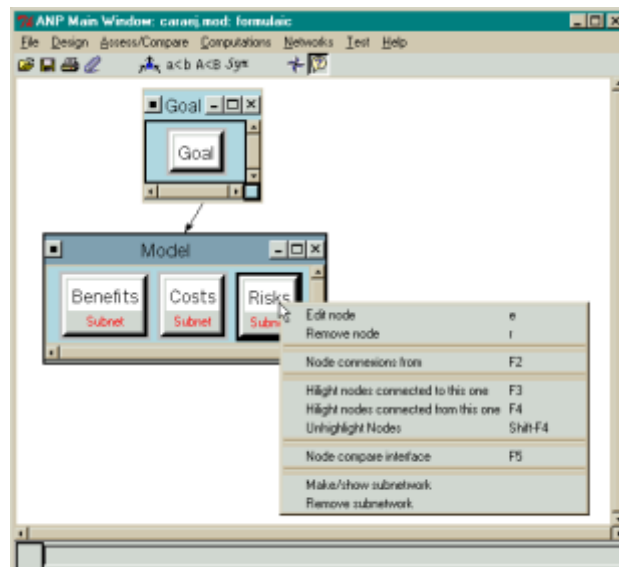


Figure 62. The Dropdown Node Menu obtained by Right-clicking with the Cursor located over a Node.

CREATING A SUB-NETWORK

It is not necessary for the car model to create any subnet as they were automatically created by the template wizard. However, if you need to create a sub-network, right-click on the node to have a subnet built for it, for example, the Benefits criterion node. Select the *Make/display sub-network* menu command to open a new window for the sub-network. Nodes with sub-networks attached to them have the word subnet displayed in red at their bottom.

Tip: It is also possible to save a subnet in a separate file and open it in a blank sub-network. This will save you time entering node names if you plan the sub-network template properly. For example, after adding the Alternatives cluster to a subnet when you are constructing your own subnets, save it, then open it in the succeeding blank subnet windows.

COMPLETING THE SUB-NETWORKS

To create the sub-network for the Benefits Control Criterion:

- Place the cursor over the Benefits criterion node in the Benefits Criteria cluster and click with your right mouse button to drop down its node command menu.
- Select the Make/show network command. The window titled Subnet under Benefits criterion appears. It will already have the alternatives cluster in it. Finish the subnet so it contains the clusters and nodes shown below:

Clusters	Nodes
Advantages	Transportation, Status, Sports, Socializing
Alternatives	American, Japanese European

Select the Design Cluster New command from the menu of the window titled Subnet under Benefits criterion and enter the name of the cluster: **1Advantages** in the New Cluster Dialogue box shown in Figure 63.

To advance from one field to the next in the New Cluster Dialog box press the <Tab> key. From this dialogue box you can:

- Enter the name of the node (spaces are allowed)
- Enter the description of the node
- Select the font for the cluster when displayed as a window
- Select the font for the cluster when displayed as an icon
- Change the background color of the cluster window
- Select an icon for the cluster by double-clicking on your selection
- You may either save and stop the cluster creation process, or create another

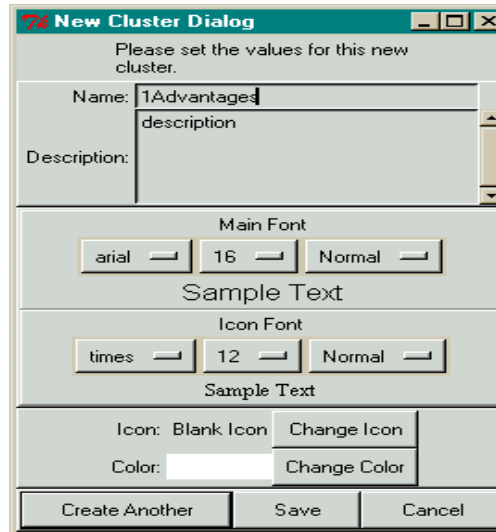


Figure 63. The New Cluster Dialog Box for Creating Clusters.

Caution: When you create clusters (and nodes) one after the other using the Create Another button they should cascade. However, if you stop the process with the Save button and later start again, the first new cluster will be created in the upper left-hand corner of the window and may land on top of a previously created cluster in that position. Left click on the top one and drag it somewhere else if this happens.

Mouse Shortcut for Creating Clusters: Press the <Shift> key and left and click with the mouse anywhere on the background of the main window. The new cluster will appear wherever the mouse cursor was at the time.

Keyboard Shortcut for Creating Clusters: Press <Shift> <n>, that is, a capital N. The new cluster will appear wherever the mouse cursor was at the time.

Once an icon has been selected for a cluster and you have saved it you can toggle between the cluster window and icon view by double-clicking with the left mouse button on the cluster or icon.

If a dialogue box disappears before you have saved, it has probably been minimized and is on the Windows menu bar at the bottom of your screen. Left-click on it to restore it and proceed with the Save.

CREATING NODES IN CLUSTERS

To create the nodes select the Design Node New command. Select the cluster into which you wish to add the nodes and type the names in the Create New Node dialog box just as you did with the clusters. Select the Create Another button to add another node until you are finished then select the button. Alternately, you may right-click on the background of the cluster into which you wish to place the nodes, and select the command Create node in cluster from the node menu that appears. The Node Dialog Box for Editing and Creating New Nodes is like the cluster dialog box, so those screen clips will not be repeated here.

Tip: Both Clusters and Nodes are organized for making comparisons and placed into the supermatrix in alphabetical order. To make the nodes appear in the order you like, preface the names with 1,2,3,....

Enter the node names in the 1Advantages cluster: 1Transportation, 2Status, 3Sports, 4Socializing. Enter the node names 1American, 2Japanese and 3European in the Alternatives cluster. The resulting completed subnet titled Subnet under Benefits criterion is shown in Figure 64 below.

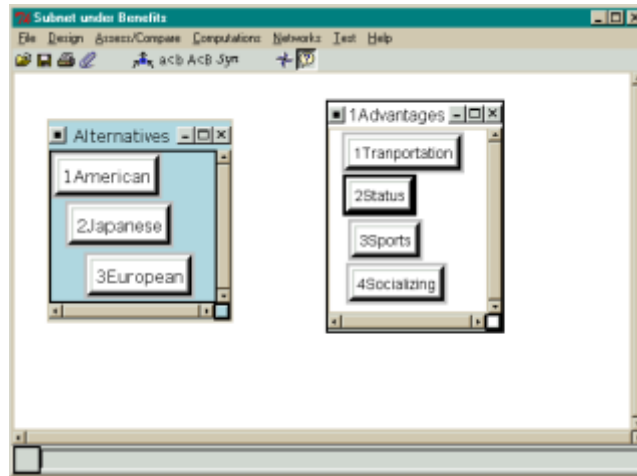


Figure 64. The Completed Subnet for the *Benefits* criterion Node.

Keyboard Shortcut: Add nodes in a cluster by placing the mouse in the cluster then pressing the letter <n>. The new node will be added where the mouse cursor is.

Another Shortcut for Adding Nodes: Select the <Shift> key and left-click over the background of a cluster to add a new node where the mouse cursor was when you left-clicked.

See the node creating/editing dialogue box below that appears when you select the command Create node in cluster shown above.

Create the Costs and Risks subnets by adding the Outlays cluster in Costs and the Bad Luck Events cluster in Risks. The model now has all the clusters and nodes in it as shown in Figure 65. The links now need to be made and the judgments entered.

We shall now describe the node dropdown menu. Click with your right mouse button on the Benefits criterion node to drop down its node menu. The commands on the menu apply to this node and are described below.

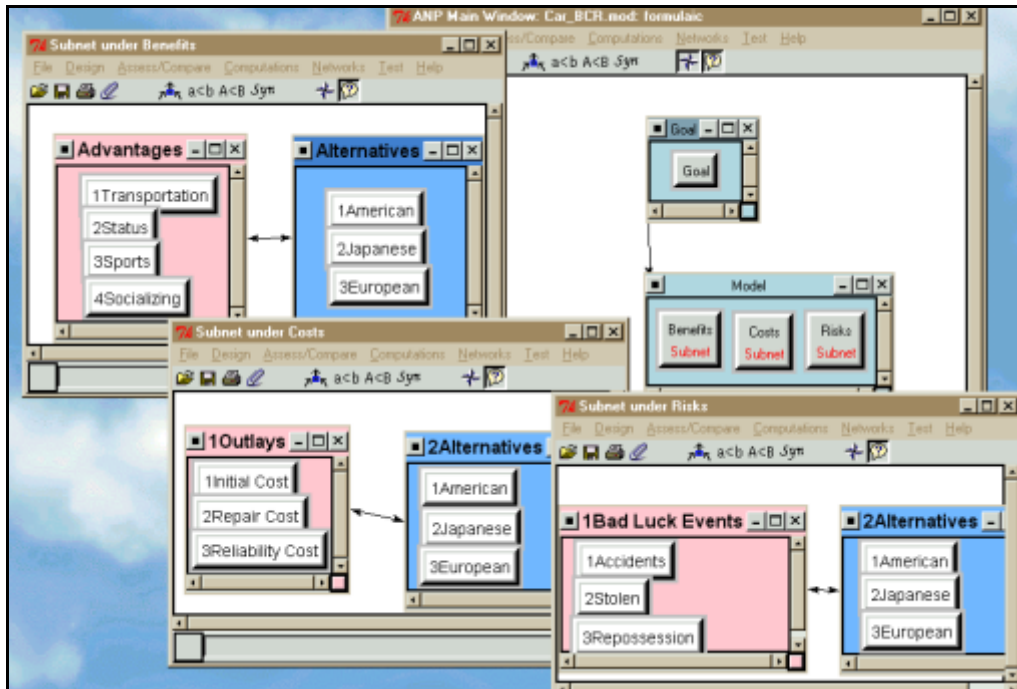


Figure 65. Completed Model: Top level and 3 Subnets.

THE NODE MENU COMMANDS

The commands in the drop down node menu are:

- *Edit node* – change the appearance of the fonts, change the title and description of the node, and select an icon for the node.
- *Remove node* – delete the node
- *Node connexions from* – create connections from this node to other nodes
- *Highlight nodes connected to this one* – highlight all the nodes in other clusters connected to this node. These would be parent nodes for comparison groups that this node belongs to.
- *Highlight nodes connected from this one* – highlight all the nodes in other clusters connected from this node. These would be nodes belonging to comparison groups for which this node is a parent.
- *Unhighlight Nodes* – turn off the highlighting that was previously turned on. This command is the only way to do that.
- *Node compare interface* – compare a group of nodes of which this node is the parent. There may be such groups in more than one cluster, so you will be asked to select the cluster the group is in. Links or connections have to be established before you can make comparisons.
- *Make/show sub-network* – create the sub-network for this node or open it if it has already been created. When first created the sub-network is an empty window. You may start a new network by creating clusters and nodes with the Design command; or you may import a network from a Word document file using the File Import command. You may also open a previously created network using the File Open command. Use the File Save command to save the new network within the current main model's file. The new network will contain all the clusters, nodes, links and judgments of the original one.
- *Remove sub-network* – this command will detach an existing sub-network from this node and it will be deleted if you do not save it or have not previous saved it under a name of its own.

CREATING CONNECTIONS OR LINKS BETWEEN NODES

The process of linking nodes is as follows. Pick a node that is a potential parent node. Look at another cluster and ask if there are nodes in that cluster you wish to compare with respect to the node you have picked. If there are, create links to them. There must be at least two nodes in the other cluster to form a group that can be compared with respect to the prospective parent node. Examine the next cluster for nodes that should be compared with respect to the parent node. Finally examine the node's own cluster to see if there is interdependence which means you want to compare other nodes in its own cluster with respect to it.

To make the connections, turn on the connection mode by clicking on this icon:



Left-click on the node that will serve as the parent and right-click on the nodes it is to be connected to.

In this example, every node serves as the parent to all the nodes in the other cluster. This is not generally the case. The links between the clusters therefore have arrows at both ends as the nodes are linked both ways. Create the links for all three subnets. To show what nodes are connected, turn on the "Show Connections" icon:



When you pass the mouse across a node, all the other nodes in the entire network that it is connected to will appear with a red outline around them. And if you have marked any comparisons that were down as completed when you exited the comparison mode, the component boxes containing nodes with completed comparisons with respect to the node your mouse is resting over will be outlined in red.

INNER AND OUTER DEPENDENCE

When elements are linked only to elements in another cluster, we say the model exhibits only outer dependence. When elements are linked to elements in their own cluster, we say there is inner dependence. There is only outer dependence in this example. All three subnets are totally connected

Notice that the Alternatives clusters contain the same nodes in all three sub-networks. Every sub-network must have a cluster named Alternatives, or some variation of the name such as 1Alternatives. The software can recognize most forms of the word Alternatives. It uses this word to synthesize the priorities for the alternatives properly. The building of the model is now complete. Next the judgments must be made.

MAKING JUDGMENTS OR PERFORMING ASSESSMENTS OR COMPARISONS

Now you are ready to begin making judgments in the model. You must go through the model selecting one node at a time and pairwise comparing or putting in data for each of the comparison groups linked to it. Make sure you have entered judgments for every parent node throughout each sub-network.

ENTERING JUDGMENTS

We will demonstrate the comparison process with the subnet for the Benefits criterion. Show the subnet by right-clicking on the background of the Benefits criterion in the control network. Select the command Make/Show subnetwork. Within the sub-network right-click on the node Status, and select the *Node compare interface* command, then select the cluster 2Alternatives containing the nodes to be compared to Status. Alternately select the *Assess/compare* command and *Node Comparisons* from the menu at the top of the Window and select the cluster containing the node you want to compare with respect to, 1Advantages,

then select the node Status. Finally select the cluster containing the nodes to be compared, 2Alternatives. The comparison window will appear for the node Status in the Questionnaire mode Figure 66.

Tip: If you will be comparing nodes in multiple clusters for the given parent node, you may select them all at once. The comparison process will then automatically cycle through offering comparisons for nodes in one cluster after another. If nodes in only one cluster will be compared, you may double-click on that cluster in the cluster selector box and go directly into the comparisons.

7% Comparisons for "2Status"

File Computations Misc. Help

Graphic Verbal Matrix Questionnaire

2Japanese is moderately to strongly more important than 1American

1. 1American	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	2Japanese
2. 1American	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	3European
3. 2Japanese	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	No comp.	3European

Figure 66. Comparisons in the Questionnaire Mode for the Alternatives under Status.

Pairwise comparisons are made between two elements in one cluster with respect to a “parent” element in another cluster. For example, the elements American and Japanese in the “Alternatives” cluster are pairwise compared with respect to Status, which is in the “Advantages” cluster. For this comparison the link is from the parent element, Status, to the elements being compared, American and Japanese. The judgment of 4 preferring Japanese to American for Status has been entered by left clicking on the 4 in the Questionnaire mode shown above. Since Japanese is preferred over American, select the 4 nearer the Japanese label on the right.

The Matrix mode for entering judgments is shown in Figure 67. These are equivalent to the judgments shown in the Questionnaire Mode. Note that the arrow next to the judgment points to the preferred member of the pair. Double-click on the button the arrow is on to invert the judgment. The button indicators at the right show which judgment you are on. In the verbal mode they are important to know how far you have come in making comparisons.

7% Comparisons for "2Status"

File Computations Misc. Help

Graphic Verbal Matrix Questionnaire

2Japanese is 4.0 times more important than th an 1American

Inconsistency	2Japanese	3European
1American	↑ 4.0	↑ 6.0
2Japanese		↑ 3.0

Figure 67. The Matrix Comparison Mode for Comparing the Alternatives with respect to Status.

The equivalent Verbal Comparison mode for comparing the alternatives with respect to Status is shown in Figure 68. Note the button at the bottom of the screen labelled "Invert Comparison". Click on it to invert the preference order.

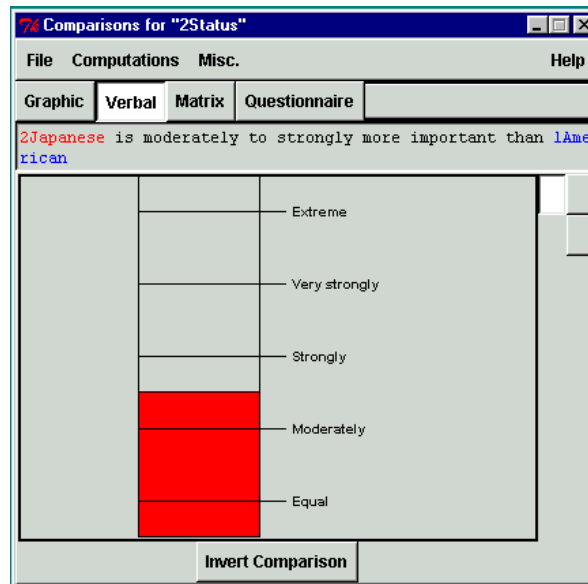


Figure 68. The Verbal Comparison Mode for the Alternatives with respect to Status.

The three carmakers will rank differently on Status depending on who is making the judgments. This is a subjective personal opinion. Different individuals have different answers and both are “right” for that individual. One’s prior experience, word-of-mouth information, and actual data all affect one’s subjective judgments. The purpose in decision-making is to come up with one’s own best choice in the sense that it is the most preferred by the individual making the decision. Suppose for the individual here that is making the judgment, Japanese carmakers are preferred to American carmakers for Status. The judgment entered is 4, equivalent to the verbal judgment of “between moderately and strongly”. The arrow on the 4 in the Matrix Comparison Mode points upward toward Japanese to indicate that it is preferred over American.

The comparative word used in the comparison phrase can be changed from the default word of importance by selecting it in the dialogue box shown in

Figure 69 below:

- Select the menu command *misc.*, *Comparison words* to bring up the dialog box
- Select a new comparative word, for example, preference.
- Or type a new comparative word of your own in the field the arrow is pointing to, "beautiful", for example.

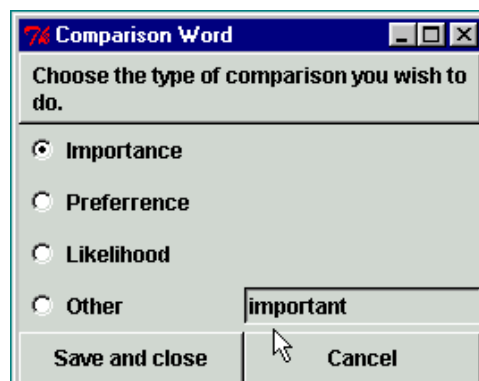


Figure 69. Dialog Box for Changing the Word used for Expressing Comparison preferences.

TIP: When making the pairwise comparisons, one should keep in mind that the control criterion is Benefits. Which of the elements being compared yields the greater **benefit** with respect to Status, American or Japanese carmakers?

The modes for making assessments are:

- Graphic
- Verbal
- Matrix
- Questionnaire
- Direct Data Entry (Select this mode under the Misc. command)

The first four modes are based on making judgments using the Fundamental Scale of the AHP from 1 to 9. Switch among these four modes by selecting the tab for the one you want. The fifth mode, Direct Data Entry is accessed under the Misc. command). In this mode the pairwise comparison entries are calculated by taking the ratio of the data for the pair being compared and entering it in the Matrix mode. If the ratio is less than 1.0, the inverse of the ratio is computed and entered as a number greater than 1 in the matrix mode with the arrow indicating which of the pair had the larger original value. All computations are based on the values currently appearing in the Matrix mode.

Tip: If after entering Direct Data you change to another comparison mode and enter one or two values in the new mode, these values will be changed in the Matrix mode while the other values will be the original ratios from the Direct Data mode. So the computations will be based on a mix of data and judgments. This is okay so long as you understand the results you are getting.

CALCULATING PRIORITIES IN THE COMPARISON MODE

To calculate the priorities based on the judgments you have entered, select the Computations, Show new priorities command in the comparison mode menu. The priorities with respect to Status are as shown in Figure 70. Priorities such as these that are the result of doing comparisons are referred to as local priorities.

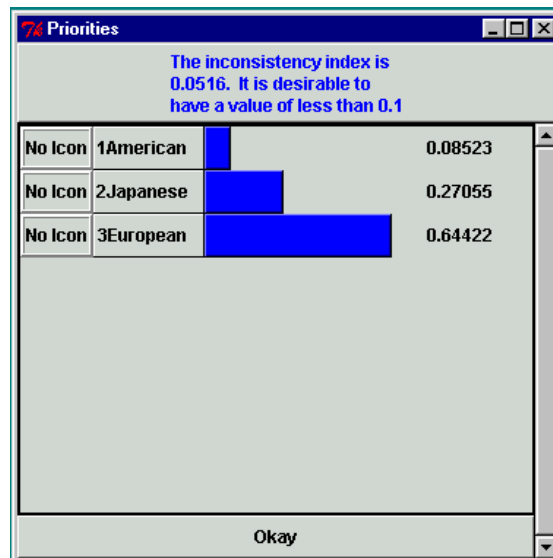


Figure 70. The Priorities for the Alternatives with

respect to Status

IMPROVING INCONSISTENCY

The inconsistency ratio is shown at the top of the Priorities screen. It is 0.0516 in this case. A ratio of less than 0.1 is desirable, so this is an acceptable level of inconsistency. If the ratio is higher, select the Computations, Most inconsistent command from the menu. Inconsistency can only be improved using the Matrix mode, so if you select that command from another mode you will be taken to the matrix mode where you can improve consistency. The most inconsistent judgment will be highlighted and you can request the Best Value for it. You can change it, or not, as you wish.

To use the suggested judgment, simply type it in the highlighted cell. The cell arrow will automatically invert to keep it correct. If a judgment such as .5 was suggested, after you type it in it will be converted and appear as a 2, with a red arrow pointing up.

THE SUPERMATRIX AND CALCULATING RESULTS

There are seven groups to be pairwise compared in the Benefits criterion sub-network because each of the elements serves as a parent node for comparing all the nodes in the other cluster. For example, Style is a parent node for comparing the Alternatives: American, Japanese and European; and American, in turn, is a parent node for comparing the Advantages nodes Transportation, Status, Sports and Socializing.

Thus there are seven local priority vectors in the Benefits sub-network and these get entered into the unweighted Supermatrix. The Computations command shown below in Figure 71 is used for the results, both in a sub-network and in a control network. It is not necessary to use the command in a sub-network, sub-network results are fed up to the control network in any case, but it is often helpful to see these intermediate results and they may be useful in analyzing the final results.

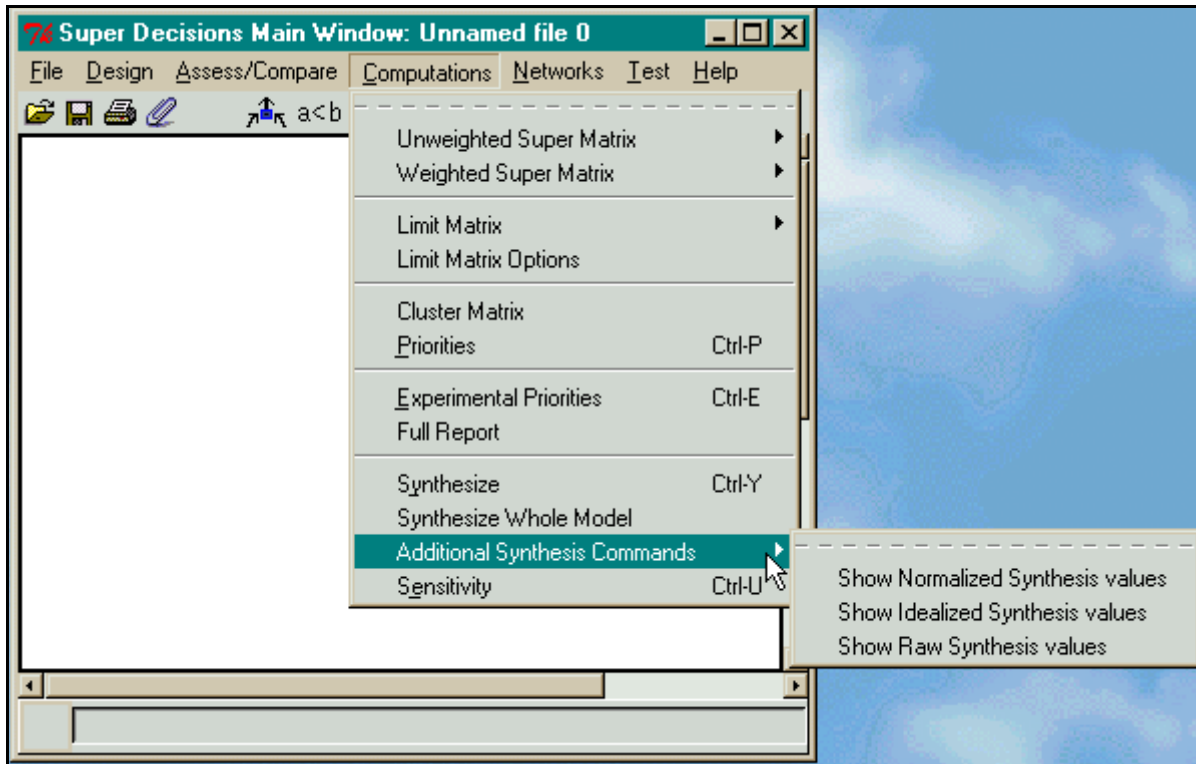


Figure 71. The Computations Menu showing the Synthesize Command

GETTING RESULTS

Here we briefly described the commands on the Computations menu:

- *Unweighted Super Matrix* – contains all the local priority vectors of the comparison groups in the network.
- *Weighted Super Matrix* – the cluster weights have been multiplied times the local priority vectors in the unweighted supermatrix to make each column stochastic (that is, sum to 1)
- *Limit Matrix* – the weighted super matrix has been raised to powers until it stabilizes - that is, all the columns in the matrix have the same values.
- *Limit Matrix Options* – these are some experimental ways to calculate the limit matrix. You should use the already selected “Calculus Option”.
- *Cluster Matrix* – The matrix of cluster weights is displayed. The number in each cell of the cluster matrix is multiplied times all the cells in the unweighted supermatrix in the corresponding component.
- *Priorities* – this command gives the priorities of all the nodes in a sub-network.
- *Experimental Priorities* is for research purposes and will soon be removed.
- *Full Report* – this command gives a very comprehensive report about the structure of the model and the various partial results, such as synthesis of alternative values from a sub-network. This report may be previewed or printed to an HTML file. You may then modify it, or extract the parts you need.
- *Synthesize* – gives the priority vector for the Alternatives in the sub-network, when the calculations are being done in the sub-network, and gives the synthesized priority vector for the Alternatives over all the sub-networks when the calculations are being done in the control network.
- *Sensitivity* – This is a what-if type of sensitivity that allows you to select any combination of independent variables. They may be nodes, supermatrix entries, or judgments. The priorities of the alternatives are graphed for each point on the x-axis of the graph, labeled “experiments”. Sensitivity

is often performed for the BOCR; it will not give meaningful results if the Multiplicative Formula is being used for synthesis, as changing the priority of any one of them will not change the answer.

We shall now show the three forms of the supermatrix in the *Benefits criterion* sub-network (remember, there are no supermatrices in the Control network). The first is the Unweighted Super Matrix in Figure 72.

Subnet under Benefits criterion: formulaic: Unweighted Super Matrix								
Cluster Node Labels		Advantages				Alternatives		
		1Transportation	2Status	3Sports	4Socializing	1American	2Japanese	3European
Advantages	1Transportation	0.000000	0.000000	0.000000	0.000000	0.489240	0.141216	0.058860
	2Status	0.000000	0.000000	0.000000	0.000000	0.065218	0.202572	0.494965
	3Sports	0.000000	0.000000	0.000000	0.000000	0.098394	0.485005	0.361038
	4Socializing	0.000000	0.000000	0.000000	0.000000	0.347148	0.171207	0.085137
Alternatives	1American	0.614411	0.085220	0.111111	0.253428	0.000000	0.000000	0.000000
	2Japanese	0.117221	0.270557	0.222222	0.084803	0.000000	0.000000	0.000000
	3European	0.268368	0.644223	0.666667	0.661769	0.000000	0.000000	0.000000
Done								

Figure 72. The Unweighted Supermatrix for the Subnet under the Benefits Criterion Node.

Subnet under Benefits criterion: formulaic: Weighted Super Matrix								
Cluster Node Labels		Advantages				Alternatives		
		1Transportation	2Status	3Sports	4Socializing	1American	2Japanese	3European
Advantages	1Transportation	0.000000	0.000000	0.000000	0.000000	0.489240	0.141216	0.058860
	2Status	0.000000	0.000000	0.000000	0.000000	0.065218	0.202572	0.494965
	3Sports	0.000000	0.000000	0.000000	0.000000	0.098394	0.485005	0.361038
	4Socializing	0.000000	0.000000	0.000000	0.000000	0.347148	0.171207	0.085137
Alternatives	1American	0.614411	0.085220	0.111111	0.253428	0.000000	0.000000	0.000000
	2Japanese	0.117221	0.270557	0.222222	0.084803	0.000000	0.000000	0.000000
	3European	0.268368	0.644223	0.666667	0.661769	0.000000	0.000000	0.000000
Done								

Figure 73. The Weighted Supermatrix for the Benefits criterion Subnet; it is the Same as the Unweighted Supermatrix in this Case.

Note that the values are exactly the same for the Weighted Super Matrix shown in Figure 73 as they were for the unweighted supermatrix. This is because no cluster comparisons were done – nor were they necessary because the columns in the unweighted supermatrix already sum to 1. It would have been possible to do cluster comparisons – even though there are only two clusters – if one of the clusters had been inner dependent and linked to itself.

The Cluster Matrix is shown below in Figure 74. No cluster comparisons were made, so it has the default values of 1.0 for the non-zero components in the unweighted supermatrix: the (Alternatives, Advantages) component and the (Advantages, Alternatives) component. And it has the default values of 0.0 for the zero components which are the (Advantages, Advantages) component and the (Alternatives, Alternatives) component.

Cluster Node Labels	Advantages	Alternatives
Advantages	0.000000	1.000000
Alternatives	1.000000	0.000000

Done

Figure 74. The Cluster Matrix for the *Benefits criterion* Subnet.

Cluster Node Labels		Advantages				Alternatives		
		1Transportation	2Status	3Sports	4Socializing	1American	2Japanese	3European
Advantages	1Transportation	0.082266	0.082266	0.082266	0.082266	0.082266	0.082266	0.082266
	2Status	0.173686	0.173686	0.173686	0.173686	0.173686	0.173686	0.173686
	3Sports	0.165716	0.165716	0.165716	0.165716	0.165716	0.165716	0.165716
	4Socializing	0.078332	0.078332	0.078332	0.078332	0.078332	0.078332	0.078332
Alternatives	1American	0.103611	0.103611	0.103611	0.103611	0.103611	0.103611	0.103611
	2Japanese	0.100104	0.100104	0.100104	0.100104	0.100104	0.100104	0.100104
	3European	0.296285	0.296285	0.296285	0.296285	0.296285	0.296285	0.296285

Done

Figure 75. The Limit Matrix for the *Benefits criterion* Subnet.

Figure 75 shows the limit matrix to which the weighted matrix converged when raised to powers. The limit matrix has been reached when all the columns are the same.

Elect the Computations Priorities command to get the view shown in Figure 76 of the priorities for all the nodes in the Benefit criterion subnet. These are simply the values from any column of the Limit Matrix. They are shown in two ways: the Limiting values are from the Limit Matrix. The Normalized by Cluster values are obtained by normalizing the priorities in each component so they sum to 1.0.

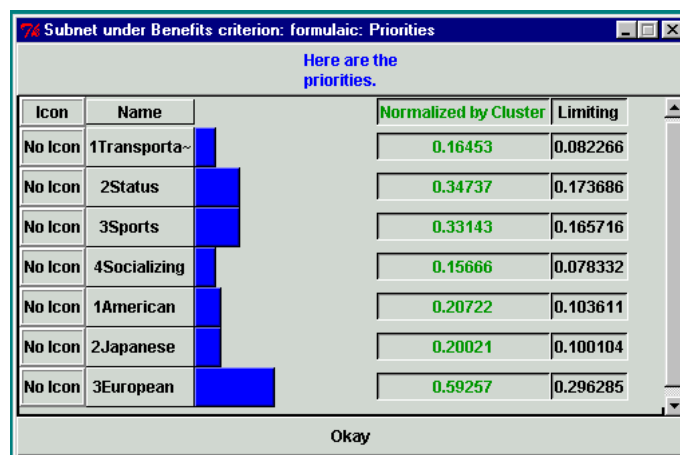


Figure 76. The Priorities for the *Benefit criterion* Subnet.

Finally, select the Computations Synthesize command to get the synthesized priorities for the nodes in the Alternatives cluster in the *Benefits criterion* subnet. They are shown in Figure 77 below. The analysis that one might apply here is that for benefits the European car is preferred, almost 3 to 1, to either the Japanese car or the American car. Why don't we all drive European cars? Well, there are drawbacks as will be seen when we examine the costs and risks subnets.

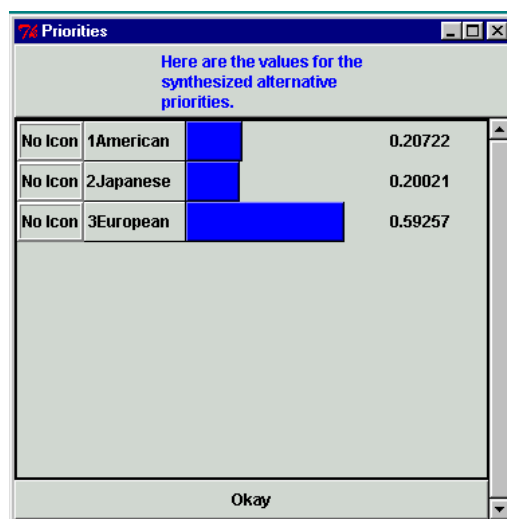


Figure 77. The Synthesized Priorities for the Alternatives in the *Benefits criterion* Subnet.

You may complete this model by making judgments in all of the subnets. After you have finished that, go to the top level Control network and select the Computations, Synthesize command to synthesize the results for the entire model as shown in Figure 78. These results show that overall the American car is slightly the best choice. Recall that the overall synthesis in the top-level control model used the additive form of the BOCR equation.

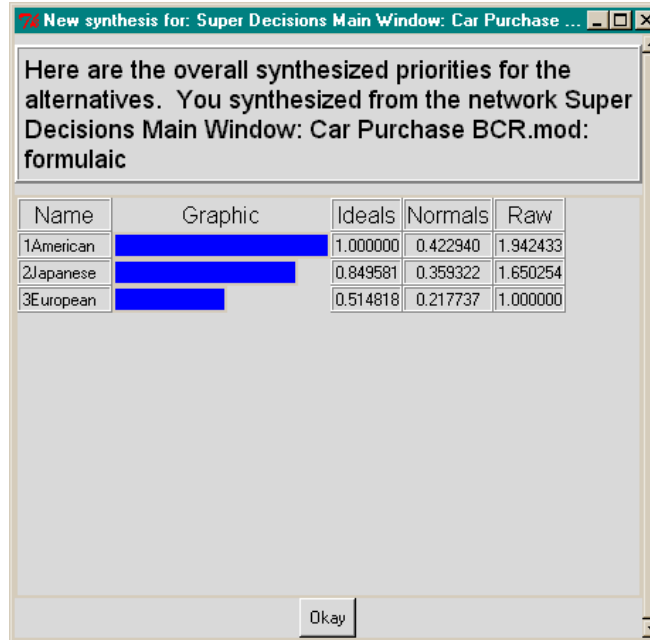


Figure 78. Overall Synthesized Priorities

Note that the Computations, Priorities command will give default equal priorities of .333 for each of the BCR nodes in the Control model, because though they are linked to the goal, no comparisons were made or distinguishing data entered as to their importance in this decision.

SENSITIVITY

To display a sensitivity graph select Computations, Sensitivity from the menu of the top level model. (You can also perform sensitivity from lower levels.) You cannot perform sensitivity for the merits of BOCR if the Multiplicative formula is selected, because the effects of changing their weights cancel out due to the formula. You must select one of the other formulas, for example, the Additive formula.

This is a what-if type of sensitivity that allows you to select any combination of independent variables. The initial sensitivity screen that appears always starts with one node selected, the first alphabetically in the network where the command was issued. The startup sensitivity screen for the Car Purchase model has the Goal Node selected as shown in Figure 79.

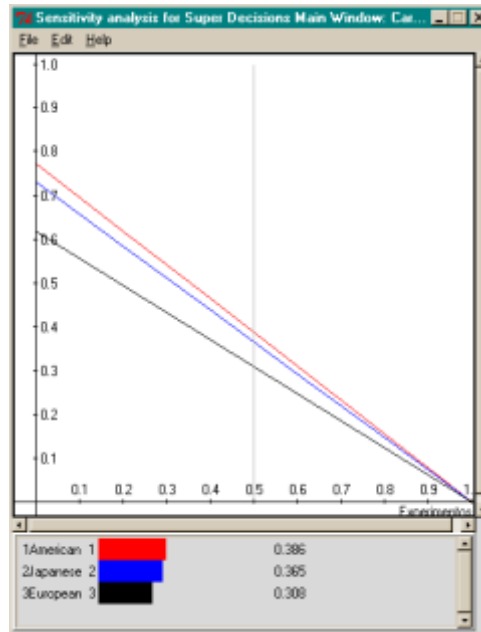


Figure 79. Startup Sensitivity Screen (with Goal as Selected Node) for *Car Purchase Model*

You need to change it to the Benefits node, by selecting the Edit, Independent Variable command on the Sensitivity analysis screen. The Sensitivity input selector will appear as shown in Figure 80.

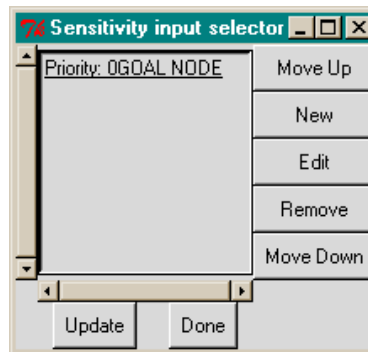


Figure 80. Sensitivity Input Selector

Remove the Goal node and select the New button to choose a new independent variable. From the New Parameter screen select the parameter type, the network and the independent variable node. You may also select Start and End values on the x-axis and the number of steps as shown in Figure 81:

<i>Parameter Type Options</i>	<i>Network selection Options</i>	<i>Wrt Node Options</i>
-------------------------------	----------------------------------	-------------------------

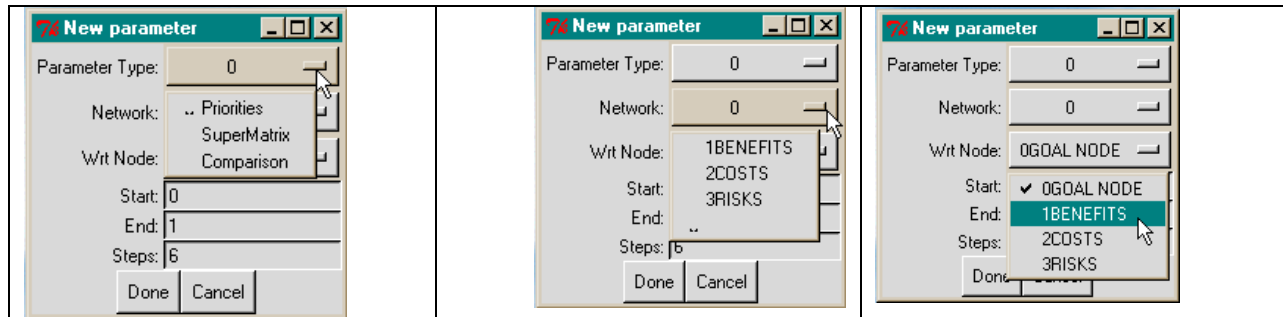


Figure 81. Selecting the Independent Variable from the New Parameter Screen.

Selecting Priorities sets the *Parameter Type* to zero. For *Network* the top-level network, network 0, is always automatically selected when this screen appears. For *Wrt Node* you need to select the node with respect to which you want to perform sensitivity, Benefits in this case. Leave the number of steps as they are, click the Done button, then click the Update button on the Sensitivity input selector screen to get the sensitivity graph shown in Figure 82 with the Benefits node selected.

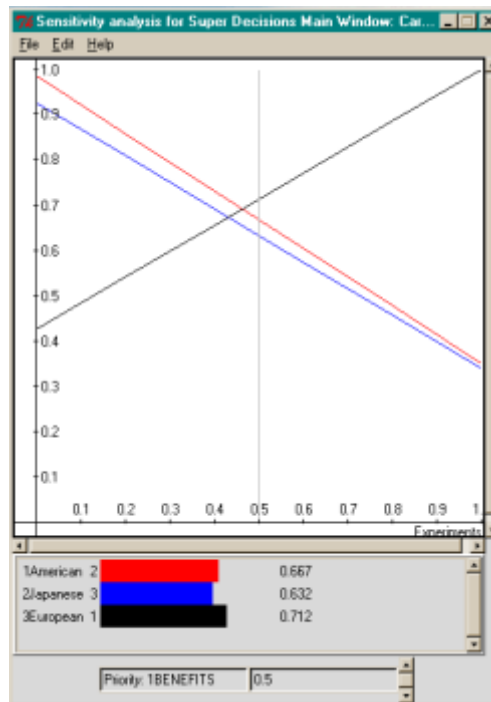


Figure 82. The Sensitivity Graph with Benefits Selected as the Dependent Variable

You may select more than one independent variable and perform sensitivity on two variables at a time. For each independent variable a range of values, and the number of steps, can be selected. Many points are generated for all the possible combinations of independent variable values. If there is a single independent variable selected, with a range of priority for that variable from .2 to .4 selected, with 4 steps (5 points of calculation) for sensitivity examination, there would be points for the corresponding alternative values plotted for the independent variable when its value is .2, .25, .30, .35, and .4. These points would be spread equally across the x-axis. The numbers on the x-axis relate only indirectly: the .30 value of Benefits would be plotted at 0.5. The lines for the alternatives are obtained by joining the points relating to a given alternative. Move the blue line back and forth by clicking and dragging and read the value of benefits and the alternative values (under the graph) for the current location of the blue line.

If there are three independent variables selected, with 4 steps, there would be 5^3 different combinations of dependent variable values plotted. These are “experiments”. The results of the 125 experiments are plotted along the x-axis (and simply joined to make them look like continuous lines). The Dimensions command is especially useful to stretch out the graph along the x-axis so you can examine more closely where the rank switches occur. Try using 800 or 1200 for the x-axis dimension.

Select Independent Variable from the Edit menu in Sensitivity Analysis to see the Sensitivity input selector shown in Figure 83. Note that the priority of Costs has been added in this figure.

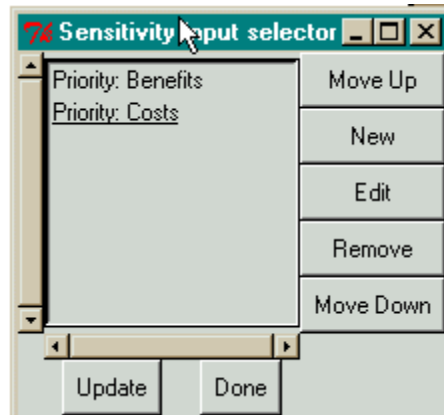


Figure 83. Sensitivity Dependent Variable Input Selector

Don’t get carried away with the number of steps you select. The calculations can get huge if you have selected too many independent variables and lots of steps. Selecting Benefits, then Costs leads to the graph shown in Figure 84.

The vertical blue line is at .5 shows the results half-way through the 27 combinations of values of Benefits and Costs that were calculated and plotted. At that particular point in the combinations, the values for the independent variables and the alternatives are shown below the graph. You may move the blue line by clicking and dragging, then read off the intersect values from the graph. You may also elect to have the corresponding values of other parameters shown by selecting the Extra Params command on the Sensitivity Analysis Edit menu.

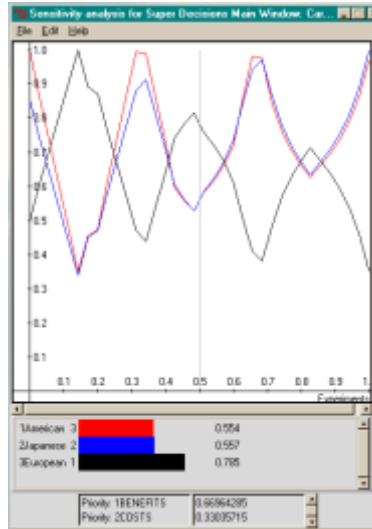


Figure 84. Sensitivity Graph with Benefits then Costs Selected as Dependent Variables

EXPORTING SENSITIVITY DATA

To export the data points of the sensitivity plot to Excel, select File Save from the Sensitivity Analysis menu. Use a name ending in .txt when asked for a file name. Then load Excel, select the File, Open command and open the file. Be sure to change the type of file to *.* to display names of all files so the .txt file will show up. The Excel import wizard will then come up. Step through it selecting OK or Next to Finish at the end. The Excel spreadsheet containing the sensitivity data points will then appear. You can use it then for displaying the data in more ways and for data manipulation – such as arranging one of the alternative columns in decreasing order, so you can see the maximum value obtained.

FILE BACKUP AND AUTOMATIC SAVING

The normal way to save is to save a complete model in a file with a .sdmod extension. You may also save parts of a model:

- 1) the current network only;
- 2) the current network and its sub-networks.

Models may have been saved from earlier versions of the software in a compressed format. Compressed models have file names with the double extension .mod.gz, or .sdmodzip. Models that have been saved in compressed format are opened using the File Open command (not by double-clicking).

Use the *File Save* command to save the currently loaded model as a file with a .sdmod extension.

Use the *File Save As* command to save the currently loaded model under a new name.

SAVING OPTIONS

- Whole Model
 - Only this network and its subnets
 - Only this network (no sub-networks of it)
-

Tip: The File Save command may be used to create a template of a subnet and start every subnet using the template. Why do this? As subnets always have the same cluster of alternatives, and often even other clusters are similar, so this may be a time-saver. To load a saved “template” file, select the File Open command in a subnet that has just been created and open the template, i.e., the saved subnet. By doing File Save and selecting the entire model, you will save the newly uploaded sub-network within the whole model. The template subnet will stay where it was and can be used again.

When sub-networks are left open and minimized, they appear on the Windows menu bar at the bottom of the screen as open applications. Simply click on any of them to restore them to full size. It is better to limit the number of open windows as this sometimes leads to crashes.

SPECIAL ANP SOFTWARE FUNCTIONS AND COMMANDS

FORMULAS

Formulas are used to synthesize results for the alternatives. Generally only the top-level network has a formula, but a formula may be entered for each network in an entire multi-layer system.

FORMULAS FOR COMPUTING SYNTHESIS RESULTS

It is not necessary to have a formula for a single network system as the results, for the nodes in the Alternatives cluster, come directly from the supermatrix of the network. A formula in the top-level network of a multi-layer system is used to combine results from the subnets for the alternatives. A formula gives the user control over how synthesis is done: additive, multiplicative and so on. If there is no formula, the synthesize command sums the results using simple addition. This is right only when there are no nodes with subnets that are “against” you such as costs and risks. There will be more about this when the various options for formulas are given at the end of this section. Synthesis results are presented in three forms as shown in Figure 85:

- **Ideal** The ideal form is obtained by dividing the raw value for each alternative by that of the largest alternative.
- **Normal** The normal form is obtained by summing the raw values for the alternatives and dividing each by the sum.
- **Raw** The raw numbers are obtained either from the supermatrix or by combining ideal values from subnets depending on the network where the synthesis command is used.
 - **Simple Network Model** If you synthesize in a model that consists of a single network with a cluster named Alternatives then the raw scores come directly from the supermatrix.
 - **Bottom Level Decision Network** If you synthesize from a bottom level decision network, the raw scores come directly from the supermatrix, because there is an alternatives cluster in the network and an alternatives component in the supermatrix.
 - **Intermediate Level Network** For intermediate networks there are no clusters named Alternatives (they are in bottom level decision networks), so alternative values are gotten only from nodes that have subnets that are feeding them up. The priorities of just those nodes that have subnets are re-computed by normalizing so they add up to one. This new priority for each node is multiplied times the ideal value for the alternatives in its subnet, and they are summed for all nodes with subnets. If the same alternative is best in all the subnets it will have a weight of one in the ideal form in the intermediate level network, but if it is not the weight will be less than one.

The rationale for doing it this way is that the most information is retained by using the ideal values, for example, an alternative with an ideal score of .75 in a decision network is 75% as good as a perfect alternative. So making the raw values in intermediate networks ideal-based retains the information about how an alternative scores against an ideal. If you use normalized values this information would be lost.

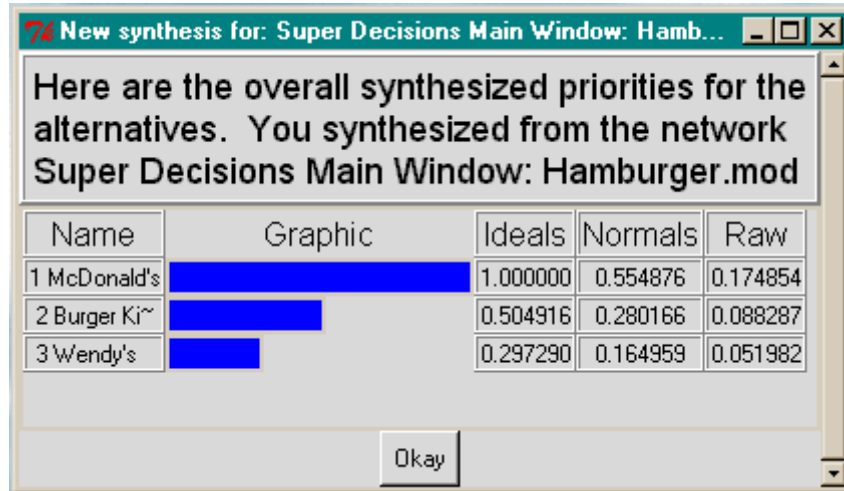


Figure 85. Example of How Synthesis Results are Displayed in each Network – from the *Hamburger* Model.

Formula commands are under the Design, Standard Formulas menu shown in Figure 86. A formula may be set using the Design, Add/Edit formula and typing a formula into the Edit Network Formula dialog box shown in Figure 87.

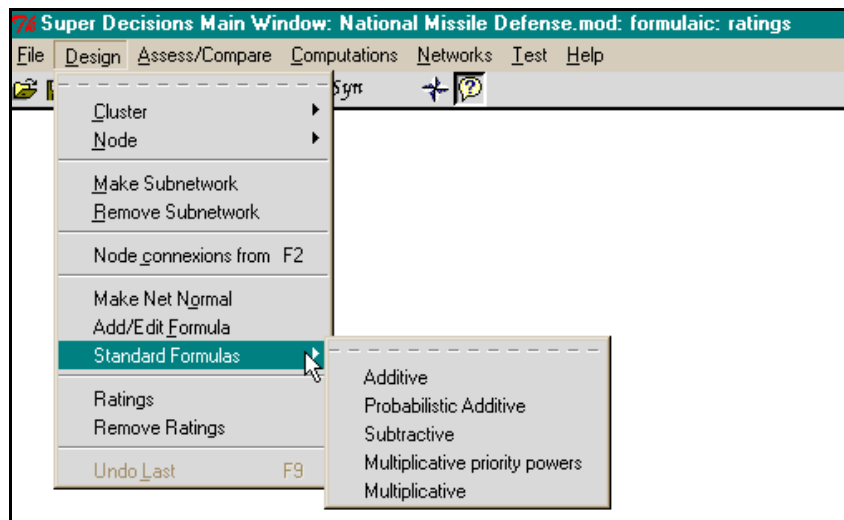


Figure 86. Commands for Editing and Selecting Formulas on the Design Menu.

Edit Network Formula

Please enter the formula and default values for this network.

Formula:

$$\begin{aligned} & \$NormalNet(Benefits)*\$SmartAlt(Benefits) + \$NormalNet(Costs)*\$SmartInvAlt(Costs) + \\ & \$NormalNet(Opportunities)*\$SmartAlt(Opportunities) + \\ & \$NormalNet(Risks)*\$SmartInvAlt(Risks) \end{aligned}$$

Defaults:

Benefits	=	0
Costs	=	0
Opportunities	=	0
Risks	=	0

Save Test Cancel

Figure 87. The Edit Network Formula Dialog Box for Editing/Showing Formula for Current Network.

The formula shown in Figure 87 is the one for the top-level network in the *National Missile Defense* model. It is the Additive Formula and was selected from the list of standard formulas under the Design menu command. The title bar of the network window has the word Formulaic on it, meaning the formula that appears in the dialog box is being used in the synthesis process.

THE STANDARD FORMULAS

You can select a formula from the Standard formula list, and you can change it at any time. If no formula is selected, it means use the Alternatives' priorities from the limit supermatrix if you have a single network model, or add the results for the Alternatives coming up from subnets for any nodes that have subnets. This formula is somewhat mindless and simply adds regardless of whether the nodes with subnets are "for" you or "against" you. Smarter formulas invert the values under nodes named Costs and Risks, so that the worst alternatives get smaller values. In general you should select a formula if you have a multi-layer model.

- Additive:** This formula uses un-inverted values for the alternative values coming up from subnets under Benefits and Opportunities and inverted values for those coming up from Costs and Risks subnets. The inverted values are "flipped" so that the most costly alternative, for example, now has the lowest priority. This is accomplished by taking the inverse of each alternative's priority. If the priority is a_1 for example, its inverse is $1/a_1$. Sum these inverses for all the alternatives, and divide each inverse by the sum. Then re-normalize. This will completely flip the priorities the other way around, and they will maintain their proportionate values. Weight the alternative values appropriately by the respective priorities of the BOCR in the top-level network, use priorities obtained from the Ratings spreadsheet if there is one, add and normalize to get the final result. The formula is:

$$\begin{aligned} & \$NormalNet(Benefits)*\$SmartAlt(Benefits) + \$NormalNet(Costs)*\$SmartInvAlt(Costs) + \\ & \$NormalNet(Opportunities)*\$SmartAlt(Opportunities) + \\ & \$NormalNet(Risks)*\$SmartInvAlt(Risks) \end{aligned}$$

- **Probabilistic Additive:** This formula treats the Costs and Risks values from the subnets something like probabilities. If a likelihood of an occurrence is p, then the likelihood of not having an occurrence is 1-p. Since the high cost and high risk alternatives have the highest priorities, using the formula one minus the value coming up from the subnet changes the highest priorities from most costly (risky) to least costly (risky), and adding is appropriate.

$$\begin{aligned} & \$\text{NormalNet}(\text{Benefits}) * \$\text{SmartAlt}(\text{Benefits}) + \$\text{NormalNet}(\text{Costs}) * (1 - \$\text{SmartAlt}(\text{Costs})) + \\ & \$\text{NormalNet}(\text{Opportunities}) * \$\text{SmartAlt}(\text{Opportunities}) + \$\text{NormalNet}(\text{Risks}) * (1 - \\ & \$\text{SmartAlt}(\text{Risks})) \end{aligned}$$

- **Subtractive:** This formula leaves the most costly and risky alternatives with the highest priorities, as they come up from the subnets, but subtracts from the benefits and opportunities. It can end up giving negative results. If resource allocation is the objective, Alternatives with negative results probably should not receive any investment.

$$\begin{aligned} & \$\text{NormalNet}(\text{Benefits}) * \$\text{SmartAlt}(\text{Benefits}) + \$\text{NormalNet}(\text{Costs}) * (-\$ \text{SmartAlt}(\text{Costs})) + \\ & \$\text{NormalNet}(\text{Opportunities}) * \$\text{SmartAlt}(\text{Opportunities}) + \$\text{NormalNet}(\text{Risks}) * (- \\ & \$\text{SmartAlt}(\text{Risks})) \end{aligned}$$

- **Multiplicative Priority Powers:** Here the alternative values coming up from the subnets are raised to the power of the BO and CR (in the main network, or from ratings if they were done that way), where the C and R powers are negative, that is the BO product is divided by the CR product.

$$\begin{aligned} & [\text{root } \$\text{SmartAlt}(\text{Benefits}) \$\text{IdealNet}(\text{Benefits})] * [\text{neg_root } \$\text{SmartAlt}(\text{Costs}) \$\text{IdealNet}(\text{Costs})] * \\ & [\text{root } \$\text{SmartAlt}(\text{Opportunities}) \$\text{IdealNet}(\text{Opportunities})] * [\text{neg_root } \$\text{SmartAlt}(\text{Risks}) \\ & \$\text{IdealNet}(\text{Risks})] \end{aligned}$$

- **Multiplicative:** This is the original BO/CR formula. The alternative values coming up from the subnets for B and O are multiplied and the result is divided by the product of the alternative values coming up from the subnets for C and R. In this case, even if the BOCR are weighted, it would not change the result from leaving them equal, as their weights would cancel out because of the formula. This is a marginal utility type of formula.

$$\$ \text{SmartAlt}(\text{Benefits}) * [\text{invert } \$ \text{SmartAlt}(\text{Costs})] * \$ \text{SmartAlt}(\text{Opportunities}) * [\text{invert } \$ \text{SmartAlt}(\text{Risks})]$$

Some definitions of the terms used in the formulas follow:

- **Alt(Node name)** refers to the alternative values in the subnet belonging to Node name
- **SmartAlt(Node name)** means to pick the best form of the alternative values to pass up: if the subnet is a bottom level one, pass up the Ideal values from synthesis in the subnet; if it is a bottom level subnet where the alternatives' values come from ratings, or if it is an intermediate level subnet, pass up the Raw values from synthesis in the subnet.
- **Net(Node name)** means to use the priority value of Node name in the current network
- **IdealNet(Node name)** means to use the ideal value of Node name in the current network
- **NormalNet(Node name)** means to use the normalized value of Node name in the current network. If Node name appears in an attached ratings spreadsheet use its normalized value from there.
- **InvAlt(Node name)** means to flip the values of the nodes in the subnet so that the node formerly having the largest value (most costly, most risky, for example), now has the smallest value (least

costly, least risky). This is done by inverting the value of each alternative, summing, dividing the inverted value by the sum and re-normalizing.

- **invert (Node name)** means to invert the value of Node name by taking $1/\text{Node name}$. It is not the same as the flipping described above.

The five formulas lead to the following synthesis results, shown in normalized form:

Alternatives	Additive	Probabilistic Additive	Subtractive	Multiplicative Priority Powers	Multiplicative
<i>Deploy</i>	.331	.333	.201	.556	.493
<i>Global Defense</i>	.291	.307	.107	.371	.379
<i>R&D</i>	.212	.223	-.193	.067	.220
<i>Termination</i>	.165	.137	-.500	.006	.018

Notice that the ranks of the alternatives are the same regardless of which formula is used, though this is not always necessarily so. The formula one picks depends on the use one wants to make of the results. If the purpose is to simply pick the best alternative, any one of the five formulas will do.

You can try out various formulas after your model is completed. Simply change the formula by selecting the new one you want and synthesize.

The **meaning of \$ when used in a formula**: The \$ in the formula means to compute the results for each alternative in turn.

SYNTHESIS RESULTS

The **synthesis results** for the alternatives can be displayed from any network or sub-network. The results for any network are obtained by synthesizing all subnets (if there are any) up through the network where the command is issued. Results are presented in three different ways:

Raw If the network is a bottom level decision network the values for each alternative are drawn directly from the Limit Matrix for that network. If the network is an intermediate network containing control criteria, the nodes that are the control criteria are normalized and multiplied times the Ideal values of the alternative vectors coming from the networks below each of them. These weighted vectors are then added to yield the Raw score vector. When the word Totals is used in a formula, it means use the Raw numbers from synthesis in the subnet below.

Normal A vector of priorities for the alternatives is normalized by adding the elements in it, and dividing each element by the sum to yield the normalized vector. The sum of the numbers in the normalized vector is 1.

Ideal The Totals priority vectors above are converted to Idealized values by dividing each element in the vector by the largest, so that the best alternative gets a priority of 1 and the others get their proper proportion less than 1.

ATTACHMENTS

Other files that relate to a model may be attached within it using the paperclip attachment icon shown below.



For example, a model to determine Ford's best strategy with respect to the Ford Explorer/Firestone Tire problem in which the tires were suspected of contributing to rollover accidents in Explorer SUV's (Sport Utility Vehicles) is shown in Figure 88. It had several associated files attached within it: a text file, a powerpoint slide show, an Expert Choice AHP model and a report document written in Word. The files become permanently attached to the model.

To launch an attached file, double-click on it. When the attached file is open, you cannot return to the *SuperDecisions* software. You must close the attached file to return to the model.

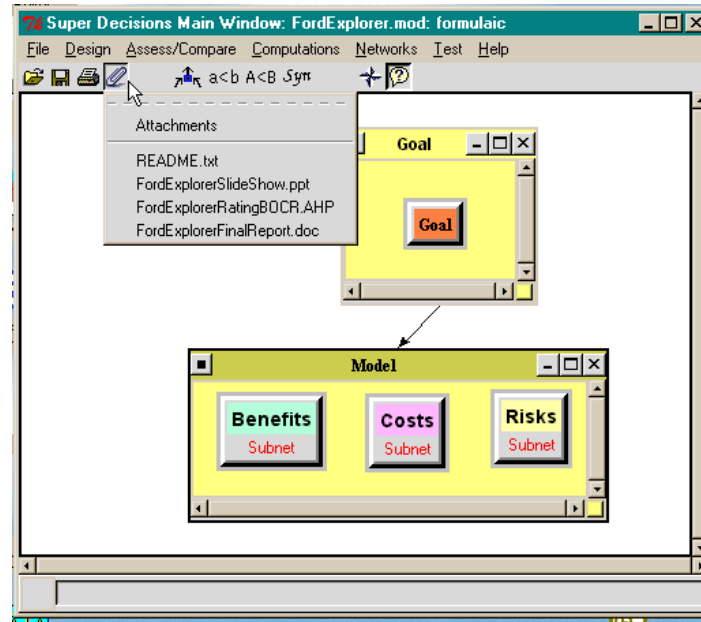


Figure 88. Attached Files within a Model

PRINTING

Printing is done from the Computations Full Report menu. One can also use screen captures. Display what you wish to print in a window or dialogue box and press <Alt> and <PrtSc> simultaneously to capture the currently active window. Then go to a Word document and select the command Edit Paste and paste the entire window into the document. It is difficult to display large supermatrices in this way. In that case it may be better to export as a text file to Excel.

Select the File Export command to export the Unweighted Super Matrix, the Weighted Super Matrix, the Limit Matrix and the Cluster Matrix as text files. You can then import them into Excel and use Excel's formatting and printing capabilities. For example, the Unweighted Super Matrix for this model will be exported to the text file: *aejcar.unweighted.txt*. To import it into Excel, launch Excel, select the File Open command and select the .txt file from the subdirectory where it is. Remember to set the Excel file type to all files (*.*) so it will appear. Then use the Excel import wizard. Just click through the wizard using the default types and the file will open as an Excel spreadsheet with cluster and element labels.

SOME CHARACTERISTICS OF SUB-NETS

- They can be different in structure, connections and factors, except for the alternatives, which must be the same.
- They may, of course, be different with regard to their judgments.
- It is critical to observe the exact spelling of the common set of alternatives.

The only requirement in building the control model relates to the final result. As the various ratios have to be computed for the alternatives across several sub-networks they need to be identified to the software by being in a cluster labeled Alternatives in each sub-network. The elements we identify as being alternatives are the focus of our concern for which we would like to know the BOCR or general results. So whenever the Computations Synthesize command is used, it gives the results for whatever is in the component we have labeled as Alternatives.

QUICK STEPS FOR DOING A COMPLEX MODEL WITH RATINGS

1. Create the top-level model with the BOCR in a cluster together. Build the strategic criteria structure for rating the BOCR; use a goal cluster with a goal node, a strategic criteria cluster with the main criteria and a cluster for each of the main criteria containing their strategic subcriteria. Pairwise compare the main criteria with respect to the goal and pairwise the strategic subcriteria with respect to their strategic criterion.
2. Create a Ratings structure attached to this network using the Build Ratings command.
 - Use the Edit Criteria command in the Ratings spreadsheet and select the strategic subcriteria you want to use to rate the BOCR;
 - Use the Edit Alternatives, New command to create the alternatives. Create alternatives named Benefits, Opportunities, Costs and Risks.
 - Create categories for each of the strategic subcriteria, (a category is, for example, Hi, Med, Low).
 - Select the comparisons button on the category creation screen and pairwise compare Hi, Med, Low to establish priorities for them. You can save and re-use categories.
 - Rate the BOCR by selecting the appropriate category in each cell in the spreadsheet.
 - Use the Calculations Totals command in Ratings to obtain the raw total scores for each of the BOCR; use the Calculations Priorities command in Ratings to normalize the raw scores. The normalized values are used for the priorities of the BOCR to do synthesis in the top-level network.
3. Create subnets for each of the BOCR. These are called the control criteria subnets.
 - Build a hierarchical structure in each subnet, with a goal cluster having a goal node, a cluster of main criteria, and a cluster for the subcriteria of each of the criteria.
 - Link the goal to the criteria, and each criterion to its subcriteria. Pairwise compare throughout for importance to establish priorities for the subcriteria. These subcriteria are your potential “control criteria”.
 - Do the Computations, Priorities command and look for the high priority subcriteria in the Limiting column. Pick some of them to become your control criteria. You should have enough of them to cover about 70% to 80% of the total priority (for the subcriteria).
4. Build subnets for each of the selected control criteria.

- These are called the decision subnets. There must be an alternatives cluster in every lowest level subnet along with other clusters. Subnets often look similar, so it is often useful to make a template at this point and reuse it as you build the rest of the subnets.
 - Link the nodes appropriately and do the pairwise comparisons throughout. In Benefits decision subnets ask the pairwise comparison question as to which is most beneficial. Similarly for Opportunities. In Risks and Costs decision subnets ask the pairwise comparison question as to which is riskiest or costliest. The worst alternative gets the highest priority for Risks and Costs.
5. Get results from the top-level model by using the `synthesize` command.
 - Set your formula using the `Design Standard Formulas` command and selecting the one you want. Start with the `Multiplicative` formula and record the results for your report; then use the `Additive` formula and the `Subtractive` formula.
 6. Perform sensitivity by selecting the `Additive` formula and do the sensitivity graph for each of the BOCR nodes. You cannot perform sensitivity using the `Multiplicative` or `Subtractive` formulas. You can also do sensitivity for other nodes, for example, control criteria nodes in the subnets. All sensitivity graphs should be done from the top-level network. Pick the node you want to do sensitivity for by selecting it using the `New Parameter` selector within the sensitivity process.

Chapter 4

APPLICATIONS OF THE ANP

1. SINGLE CONTROL CRITERION MODELS

In this chapter we give a few illustrations of the ANP. We begin with a simple example about managing the water level in a reservoir. This example is somewhat analogous to the car example that we used to describe how to use the software. We then give a more elaborate example predicting market share in the hamburger industry. The next two applications are examples of complex decisions involving multiple control criteria in a BOCR setting and were award winners. The first is choosing the best strategy for the University of Pittsburgh Medical Centers (UPMC) and the second with the best policy that China would have to follow to maintain the leading financial status for Hong Kong in East Asia.

EXAMPLE 1. WATER RESERVOIR EXAMPLE – OPTIMAL LEVEL OF WATER IN A DAM

Here we are faced with the decision to choose one of the possibilities of maintaining the water level in a dam. Levels can be Low (L), Medium (M) or High (H) depending on the relative importance of Flood Control (F), Recreation (R), and the generation of Hydroelectric Power (E), respectively, for the three levels. As in the car selection example, the first set of three matrices gives the prioritization of the alternatives with respect to the criteria and the second set, those of the criteria in terms of the alternatives. As is usual in this book, the eigenvector is the priority vector. Its derivation is discussed later in this book.

Which level is best for flood control?

Flood Control	Low	Med	Hi	Eigenvector
Low	1	5	7	.722
Medium	1/5	1	4	.205
High	1/7	1/4	1	.073
Consistency Ratio	.107			

Which level is best for recreation?

Recreation	Low	Med	Hi	Eigenvector
Low	1	1/7	1/5	.072
Medium	7	1	3	.649
High	5	1/3	1	.279

Consistency Ratio .056

Which level is best for power generation?

Hydroelectric Power	Low	Med	Hi	Eigenvector
Low	1	1/5	1/9	.058
Medium	5	1	1/5	.207
High	9	5	1	.735

Consistency Ratio .101

At Low Level which attribute is satisfied best?

Low Level Dam	F	E	R	Eigenvector
Flood control	1	3	5	.637
Recreation	1/3	1	3	.258
Hydroelectric power	1/5	1/3	1	.105

Consistency Ratio .033

At Intermediate Level which attribute is satisfied best?

Intermediate Level Dam	F	E	R	Eigenvector
Flood control	1	1/3	1	.200
Recreation	3	1	3	.600
Hydroelectric power	1	1/3	1	.200

Consistency Ratio .000

At High Level which attribute is satisfied best?

High Level Dam	F	E	R	Eigenvector
Flood control	1	1/5	1/9	.060
Recreation	5	1	1/4	.231
Hydroelectric power	9	4	1/6	.709
Consistency Ratio			.061	

The six eigenvectors were then introduced as columns of the following supermatrix which represents the impact of the elements on the left on those on the top.

	F	R	E	L	M	H
F	0	0	0	.637	.200	.060
R	0	0	0	.258	.600	.231
E	0	0	0	.105	.200	.709
L	.722	.072	.058	0	0	0
M	.205	.649	.207	0	0	0
H	.073	.279	.735	0	0	0

Here one must ensure that all columns sum to unity exactly. The final priorities for both, the level of the dam and for the criteria were obtained by multiplying this matrix by itself several times until the columns stabilize and become identical in each block.

Stabilization is obtained after a few multiplications. We have

	F	R	E	L	M	H
F	0	0	0	.241	.241	.241
R	0	0	0	.374	.374	.374
E	0	0	0	.385	.385	.385
L	.223	.223	.223	0	0	0
M	.372	.372	.372	0	0	0
H	.405	.405	.405	0	0	0

In the top right block we can read off the overall priority of each of the three criteria from any column, and read off the overall priorities of the three alternatives from any column of the bottom left block. It is clear from this analysis that for the kind of judgments provided, there is preference for a high dam with priority .405 and for hydroelectric power generation with priority .385.

EXAMPLE 2 – HAMBURGER EXAMPLE RE-VISITED

This is an elaboration of the original Hamburger application to predict the market share for the big three companies in the hamburger fast food industry: McDonald's, Burger King and Wendy's. The details are not given in this writeup, but you may refer to the *BigBurger.sdm* model that is included among the sample models of the *SuperDecisions* software (go to Help, Sample Models to find it). The main purpose of this example is to show the richness of the types of clusters that may be considered in a single network model.

Figure 89 below shows the connections between clusters, that is, at least one element in each of these clusters is connected to some element in another cluster (the elements themselves are not shown, only the clusters). Except for the customer group cluster, inner dependence exists for the other clusters.

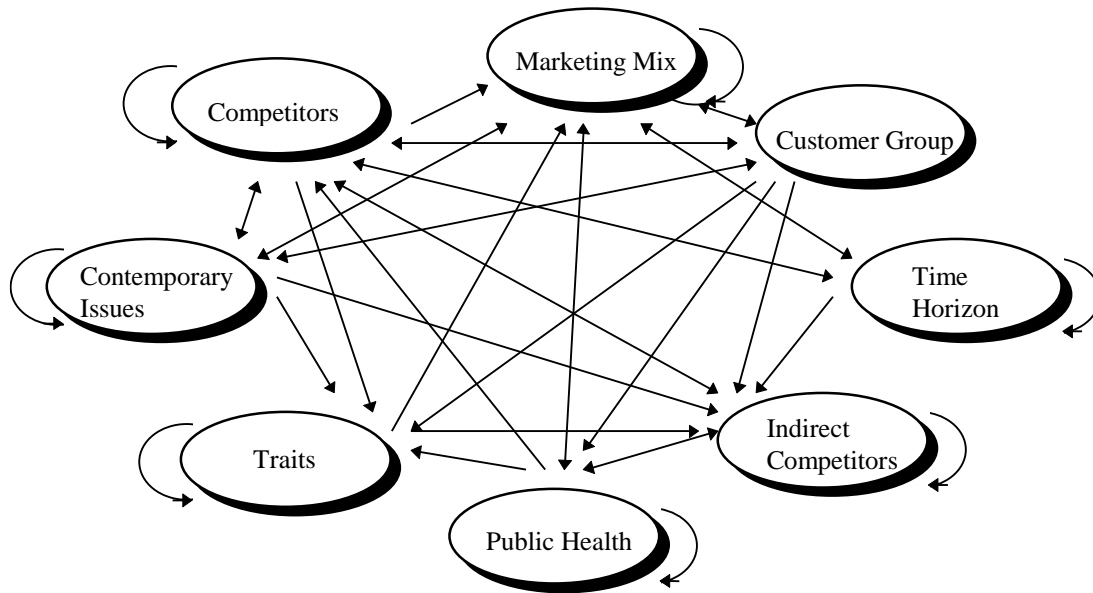


Figure 89. Overall Goal: Market Share of Competitor Group

DIRECT COMPETITORS CLUSTER

The big three companies - McDonald's, Burger King and Wendy's - are elements in the *Competitors* cluster. Each is a significant competitor with the other competitors warranting continuous monitoring and quick responses. Pairwise comparisons allow us to evaluate the importance of the two competitors with respect to their influence on each one's market share.

INDIRECT COMPETITORS CLUSTER

These are companies offering alternatives to hamburgers but competing for the same overall customer base. They include: subs (Miami, Subway, corner sandwich), fried chicken (KFC, BC), Pizza (Pizza Hut), Mexican (Taco Bell), Chinese, Steak (Ponderosa), and Diners (Full service and formal). Companies in this cluster compete indirectly against the big three by offering customers alternate foods and tastes and like the direct competitors, also compete and influence one another for market share.

CUSTOMER CLUSTER

Four basic consumer groups considered include: the white-collar professional, the blue collar laborer, the student and the family. These segments help us evaluate the influence other elements may have on each of them. For example, price may influence each segment differently. Students and families on a tight budget may be more concerned with price but the working individual may instead be more concerned with convenience. These customer segments are perceived not to influence one another as indicated by being the only cluster without inner dependence.

MARKETING MIX CLUSTER

Price in this model refers to the average price of the typical product, for example, the price of a Big Mac. However, the model could be extended to include other products and their prices. The typical product is

that which the company sells the most. For location, we consider the number of established outlets, by promotions, we mean specially packaged lunch deals that would normally be slightly more expensive when bought separately.

CONTEMPORARY CONCERNS CLUSTER

Contemporary cluster includes issues the public is aware of about the fast food industry. For example, CNN raised questions on the nutritional value of fast foods in a news report. Also, environmental groups pressure companies into paying more attention to preserving Nature and the environment by practicing recycling, properly disposing of waste, and by not over-packaging their product. These factors help raise the cost and change the routine of doing business but may also attract more customers.

PUBLIC HEALTH CONCERNS CLUSTER

Periodic outbreak of meat contamination always serves to create panic as to the safety of the meat supply channels and the adequacy of regulation and inspection. Consumers have also become highly sensitized to other evidence of hygiene related to for example, the site (clean tables, floors) and the personnel (uniforms, hats, gloves, and the handling of money together with food).

TRAITS CLUSTER

The elements consist of attributes customers may use or recall when judging one eatery over another. They are 1) the speed of service, 2) available seating and parking, 3) whether there is a delivery service and 4) the presence or absence of a drive through facility.

TIME HORIZON CLUSTER

The model makes managers think about the short and medium term measures to improve market share by connecting other elements to this cluster.

STRUCTURE

The structure of the model is described by its clusters and elements, and by the connection between them. These connections indicate the flow of influence between the elements. The convention is that an arrow going from a parent node to the children nodes indicates the children influence the parent. For example, does nutrition or packaging influence market share gained by promotion more; and if so, by how much. In other words, given a limited budget, the company has to prioritize spending on promoting one message over others. The importance of this comparison is the basis for connecting Promotion (in the *Marketing Mix* cluster) to elements in the *Contemporary Issues* cluster (packaging, nutrition, waste disposal and recycling). The reverse connection is also important because management is aware of themes in the *contemporary* cluster influence elements in the *marketing mix* differently. For example, using more costly materials that can be recycled, may raise prices more than the promotion of this fact to the public may bring in new business. Through this process of analyzing dependencies, the prevailing understanding of the marketplace is mapped out in the ANP model.

Paired comparisons are needed for all the connections in the model. For example, Burger King is connected to elements in the *Customer Group* cluster. There would be a set of numerical judgments and the derived priority weights from these judgments, represented in the reciprocal matrix shown in Table 25 below.

Table 25. Pairwise judgments of the Customer Group for Burger King

Burger King	White collar	Blue Collar	Student	Family	Priorities
White collar	1	4	5	1 / 3	0.299
Blue collar	1 / 4	1	4	1 / 3	0.138
Student	1 / 5	1 / 4	1	1 / 7	0.051
Family	3	3	7	1	0.512

The judgments in the first row of this reciprocal matrix say that in considering the market share of Burger King, White Collar workers are four times more important than Blue Collar workers, White Collar workers are 5 times more important than Students but only a third as important as Family.

The derived priorities in the last column are computed by raising the reciprocal matrix to arbitrarily large powers and then normalizing their row sums. Each priority vector entries sum to unity and are placed in their corresponding location in the supermatrix before raising the supermatrix to powers.

THE SUPERMATRIX

A supermatrix is a two-dimensional matrix of elements by elements. The priority vectors from the paired comparisons are placed in the appropriate column of the supermatrix. As the supermatrix is built in this way the sum of each column corresponds to the number of comparison sets. If Burger King only had the above two comparison sets, then the column would sum to 2 because each priority vector sums to 1. We now need to establish priorities for the clusters as they impact each cluster according to market share as shown in Table 26. The weights in the column corresponding to the cluster are multiplied by the weight of the cluster. As a result each column of the supermatrix sums to one. The resulting matrix is column stochastic.

Table 26. Cluster Weights for the Hamburger Model

	Compet.	Cust. Groups	Mktg Mix	Contemporary
Competitors	0.169	0.200	0.151	0.222
Cust. Groups	0.186		0.180	0.222

Mktg Mix	0.139	0.181	0.162	0.201
Contemporary	0.103	0.113	0.097	0.127
Pub. Hlth	0.167	0.163	0.170	
Traits	0.074	0.113	0.071	0.101
Ind. Compet.	0.162	0.229	0.106	0.127
Time Horizon			0.064	
	Pub. Hlth	Traits	Ind. Competit.	Time Horizon
Competitors	0.249	0.252	0.193	
Cust. Groups	0.175	0.252	0.178	0.391
Mktg Mix	0.157	0.218	0.112	0.195
Contemporary			0.092	
Pub. Hlth	0.220		0.218	
Traits	0.088	0.109	0.083	
Ind. Compet.	0.112	0.169	0.125	0.276
Time Horizon				0.138

The predicted market share is obtained in the row for each firm in the limiting supermatrix. These predictions and the actual market shares as appeared in the Market Share Reporter (Darney and Reddy, 1992) are presented below in Table 27.

Table 27. Predicted and Actual Market Shares for Direct Competitors

Table 3.		
	Market	Share
<u>Company</u>	<u>Predicted %</u>	<u>Actual %</u>
McDonald's	62.9	58.2
Burger King	23.9	28.6
Wendy's	13.2	13.2

We also found for the indirect competitors the results shown in Table 28.

Table 28. Predicted and Actual Market Shares of Indirect Competitors

	Market	Share
<u>Company</u>	<u>Predicted %</u>	<u>Actual %</u>
PIZZA	33.7	37.0
CHICKEN	26.0	28.4
MEXICAN	15.2	22.8
SUBS	25.0	11.7

VALIDATION

The best way to validate a theory based on judgments is to apply it to cases where the numerical outcomes are known. Use judgment to represent relations and their intensities and show that the overall process obtains results that are similar or close to the known amounts. Otherwise, applying the theory to a qualitative description of the outcome produces results which can or cannot correspond closely to the verbally described outcome. And the problem will be which is more at fault, the known qualitative outcome or the process itself and the way it combines the judgments.

This example shows that it is possible to use our experience and understanding to estimate what would actually happen in the real world. In addition, by using sensitivity analysis, we can test what needs to be done to succeed. A marketing manager can both convince himself of the problem and understand the influences of his activities on market share.

2. COMPLEX NETWORK MODELS

Now we turn to the multilevel models consisting of a top level BOCR network, control criteria networks, and decision networks containing the alternatives at the bottom.

EXAMPLE 3. THE FUTURE OF THE UNIVERSITY OF PITTSBURGH MEDICAL CENTER: STRATEGIC PLANNING WITH THE ANP

by Daniel Saaty, Joseph Charlson, Beth Clauss, Young-Chi Lin, & Ron Markle, Katz GSB

This paper summarizes the research and analysis used to determine the best strategy(s) for the University of Pittsburgh Medical Center (UPMC) to implement in order to become more competitive in a managed care environment. The study employs the Analytic Network Process (ANP) feedback modeling technique to analyze the benefits, costs and risks of five major strategic options available to UPMC. The ANP tool incorporates the interdependent influences that often exist between systemic variables. The findings suggest that UPMC should concentrate most on improving patient satisfaction and thereby the number of patients. UPMC should also develop a primary care network so that the hospital complex can attract more patients through referrals from affiliates.

INTRODUCTION

The object of this analysis is to use the Analytic Network Process (ANP) to determine the best strategies that the University of Pittsburgh Medical Center (UPMC) can implement to compete more effectively in a managed care environment. The ANP involves feedback analysis described in detail in Saaty, 1996, and would take most of this paper to summarize. This paper must assume knowledge of the ANP by the reader. This analysis focuses on strategies related to the current organizational goals of UPMC and the changing healthcare environment. The research uncovered five potential strategies that UPMC can implement individually, or in combination, to yield greater market share and profitability.

The study consists of three phases. The first involves problem definition and data collection. The second involves construction of separate feedback networks for the benefits, costs and risks of the strategies and interacting elements. The third phase involves applying the knowledge gained from the research to make judgments that define the feedback influences among the factors involved in UPMC strategic planning. From these judgments, strategies were identified for UPMC to pursue to improve the market share of the hospital complex.

EVOLUTION OF MANAGED CARE

Since the 1960s the healthcare industry has gone through tremendous change in the way care has been provided and the means by which it has been paid. Traditionally, medical care was offered by hospitals and physicians with individuals or their insurance companies covering the cost of healthcare. This traditional medical payment system is referred to as fee-for-service or FFS. Under FFS, physicians and

hospitals were paid for each service rendered and clients could choose their own provider. Insurance companies covered benefits under standard indemnity plans and exerted no strong limitations on the extent of services physicians or hospitals performed, or on the amounts charged. Interestingly enough, under the traditional indemnity plans, insurance companies did not usually cover preventative medical care.

Although its popularity has significantly declined in recent years, the FFS system is still enjoyed by many consumers and physicians. Individual consumers have their choice of doctors, and doctors can prescribe a gamut of services. The FFS system has lost popularity because insurance companies have no systems in place to prevent physicians from running up the costs for the services they prescribe. In the mid-1960s, insurance premiums began to rise significantly. This trend has continued at exponential rates in the 1990s. Another concern is that individuals who are not covered by group health plans are responsible for their own healthcare costs. In an era of uncontrollable costs, uninsured individuals may not receive any healthcare, a concern addressed by the Clinton administration.

In an attempt to bring down the costs of covering their employees' health insurance premiums, employers began to require higher contributions from employees to cover the cost of their healthcare benefits. These contributions came in the form of deductibles and percentage co-payments. Companies, realizing such contributions were not going to substantially control the rising cost of coverage, began using Health Maintenance Organizations (HMOs) to provide healthcare to their employees. Employers continue to press insurers to find more cost effective alternatives in providing healthcare to employees.

Health providers are turning to recruitment methods that target healthier employees while at the same time avoiding employees and retirees on disability programs or in long term nursing care facilities. Along with the cost-cutting measures used by many HMOs come questions regarding the potential tradeoffs that exist between managed care methods and the quality level of healthcare provided. In 1994, for example, a California business group published consumer satisfaction rating. Seventeen healthcare providers were graded from C- to A+ on services including doctor treatment and disease counseling. Most of the providers received B's and C's. Employees are given access to the ratings in choosing a provider. (*New York Times*, May 31 1995) "Health Maintenance Organizations never say they have the best heart transplant doctors in the country" (*New York Times*, May 31, 1995) In any event, employers and their insurance companies are pressuring healthcare providers to offer cost effective care without cutting quality corners.

MANAGED CARE: A DEFINITION

In 1993, the skyrocketing price of healthcare came to the forefront of legislative discussions in the US. Although no formal changes were made to the laws surrounding healthcare in the US, reform of the healthcare system had already begun as a result competitive forces.

One of the forces that is increasingly helping to control escalating healthcare costs while maintaining the high expectations of healthcare delivery in the US is managed care. Managed care is an organized system which combines the delivery and financing of healthcare. Listed below are some of the major characteristics of a managed care system:

- ❑ *Each patient must have a primary care physician or a "gatekeeper."* A gatekeeper oversees the patient's entire compendium of medical problems or risk factors, maintains a long-term relationship with the patient, and is clinically competent to know when to refer a patient to a specialist.
- ❑ *Focus on preventative care not curative care.* Since the costs of curative/specialty care are so great, managed care emphasizes the prevention of health problems and provides financial incentives to hospitals and physicians to implement this concept. The financial incentives are carried out through the way payments are made to hospitals and physicians.

- ❑ *Payments to hospitals and physicians.* Eventually, managed care companies will pay providers a fixed payment per patient per month in exchange for efficient delivery of healthcare to all patients enrolled with the managed care company. Therefore, the provider retains the difference between the amount of fixed payments received from the managed care company and the cost of providing healthcare services. The effect of this payment methodology is to give incentives to hospitals and physicians to keep patients healthy by focusing on preventative care.
- ❑ *Review of physicians' and hospitals' medical decisions.* A review board, established by the managed care company, will examine physicians' and hospitals' medical decisions to ensure they are consistent with the goals of managed care as discussed above.

It is likely that in the near future, almost every consumer of healthcare will be a member of a managed care plan. For this reason, hospitals in the Pittsburgh area must quickly determine how they are going to prosper in this new environment. Through discussions with Roger Hunt, Senior Vice President of the Northwestern Health care Network (a network of ten Chicago hospitals), Beaufort Longest, Health Administration Professor at the University of Pittsburgh, and Norman Hummon, Organizational Sociologist at the University of Pittsburgh, we defined five major strategies that are implemented have been implemented in other university hospitals in the US. The strategies are all oriented towards making university hospitals more competitive in the managed care environment. Brief descriptions of each strategy are listed as follows:

- 1) Internal Cost Reduction/Efficiency Improvement. This can be accomplished by reducing the number of administrators, managers, and/or staff. It can also be done by eliminating services or closing facilities. But the problem is that hospitals must find a way to cut overhead and at the same time become more efficient and deliver high quality care. Those that are able to accomplish this task will have the advantage of gaining contracts with managed care companies and in negotiating rates of payment.
- 2) Develop a Primary Care Network. The greater the number of primary care physicians that a hospital contracts or affiliates with, the greater the number of referrals it can receive. Since primary care physicians will act as gatekeepers, this will be the only way to get a large number of patients in your hospital.
- 3) Improve Outcomes. Although it is in the early stages of development, hospitals are beginning to measure how effective they are in delivering care. An example of a measurement of an outcome could be: the average number of times a patient needs follow-up care after surgery. The better the outcomes of a hospital, the more likely an insurance company is to develop a contract with that hospital.
- 4) Re-negotiate insurance contracts to a capitated arrangement. Recalling the definition of capitation, the insurance company will pay a fixed payment per patient per month to the provider. If a hospital has embraced the managed care concept of preventative care, then pro-actively seeking capitation arrangements with insurers allows them to capitalize on their delivery methods and gain a competitive advantage.
5. Teach Primary Care not Specialty Care. Historically, university/teaching hospitals have focused their academic efforts on specialty care. However, as managed care shifts the focus of healthcare toward primary care, teaching hospitals must react by changing their teaching methods. Also, teaching hospitals hire a large amount of their residents to their full-time staff. If teaching hospitals teach primary care, they will be able to hire primary care physicians and help attain the objective developing a primary care network.

BACKGROUND OF UNIVERSITY OF PITTSBURGH MEDICAL CENTER

UPMC is composed of research and teaching based facilities: Presbyterian Hospital, Montefiore Hospital, Western Psychiatric Institute and Clinic, the Pittsburgh Cancer Institute, Falk Clinic, the Eye and Ear Institute, and the Schools of Health Sciences. UPMC has traditionally focused on curative care rather than preventative care. Alternately, the managed care environment, which is rapidly becoming the dominant form of healthcare in this country, stresses preventative care and tries to keep specialty services and higher cost procedures to a minimum. UPMC now faces the problem of determining the best strategy to cut costs and attract more patients in order to increase revenues.

Teaching hospitals across the United States have traditionally been funded by federal moneys. But current political pressures threaten the financial stability of teaching hospitals. Under the Republican budget cuts, funding for teaching hospitals could fall between 30% and 60%. Private insurers and managed care groups are forcing down healthcare rates and refusing to pay for training and research that does not directly benefit plan enrollees. (*New York Times*, May 15 1995)

A July 1993 survey in *US News & World Report* ranked the University of Pittsburgh Medical Center among the best in the following specialties: AIDS, cancer, cardiology, endocrinology, geriatrics, neurology, orthopedics, otolaryngology, and rheumatology. The magazine also cited that more than 100 UPMC specialists were listed among the best doctors in America. In 1993, UPMC had approximately 32,000 medical/surgical admissions. Of these, 48% came from outside of Allegheny County, where UPMC is located. About 15% came from out of the state.

The UPMC organ transplant program is the world's largest and busiest. One transplant is performed every 12 hours. The medical center performs the most liver and lung transplants, and implants more artificial hearts than any other institution in the United States. Approximately 60% of all inpatients (with the exception of intensive care units) are surveyed upon discharge. In addition, all patients in the following areas are surveyed: Emergency Department, Same Day Surgery, Intensive Care Units, and Ambulatory Care. On a five point scale from excellent to poor, the majority of areas within UPMC receive a "4", or "very good" rating (*The Best Hospitals in America*, 1993).

The University of Pittsburgh Medical Center is involved in 1,299 research projects totaling more than \$599 million. Through research at the McGowan Center for Artificial Heart and Lung Research, patients with end-stage heart disease are implanted with temporary heart assist devices as a bridge to the heart. UPMC patients are admitted either through physician's referrals, preferred provider organizations, the Emergency Department, or through Falk Clinic, the UPMC's outpatient facility. Patients without a physician will have one from the hospital staff assigned to them.

According to HCIA, Inc., a company that offers the data base of the national hospitals, included financial information(See Exhibit 1). We analyzed the financial status of UPMC by the data provided by HCIA. Indicators from 1 to 10 were ranked on the basis of the decile that the indicator fell into when compared with all hospitals in its state. A rank of "1" designates the top decile and presents the most favorable position for a hospital.

In Exhibit 1, the column of efficiency is measured by case mix- and wage-adjusted expense per equivalent discharge from the acute care unit of each hospital. Equivalent discharges are computed as total hospital discharges multiplied by an outpatient adjustment factor; this factor is calculated as the ratio of gross patient revenue to inpatient revenue. Profitability is measured by computing each hospital's total profit margin. Total profit margin is calculated as the difference between "total revenue" and "total expense," divided by "total revenue."

Exhibit 1. Financial Comparison:

Hospitals	Outpatient Revenue%	Operating Revenue	Profitability	Efficiency
Allegheny General Hospital	16	389.2	4	9
Children's Hospital of Pittsburgh	28	166.5	6	1
Forbes Health System	25	127.0	9	5
Harmarville Rehabilitation	12	38.6	6	1
Jefferson Hospital	28	125.8	8	8
Magee-Womens Hospital	23	139.9	5	5
Mercy Psychiatric Institute	14	198.8	5	8
Montefiore University Hospital	23	204.6	7	10
North Hill Passavant Hospital	35	73.2	6	4
Presbyterian University Hospital	16	421.3	7	10
Shadyside Hospital	13	205.7	6	9
The South Side Hospital	29	41.2	2	7
St. Clair Memorial Hospital	28	86.9	7	7
St. Francis Medical Center	16	183.0	6	9
St. Margaret Memorial Hospital	27	86.5	4	9
The Western Pennsylvania Hospital	18	217.3	6	9
Western Psychiatric Institute & Clinic	7	171.0	8	1

Source: HCIA, *Profiles of U. S. Hospitals*, c1994

From Exhibit 1, we find that the three hospitals of UPMC, Montefiore, Presbyterian, and Western Psychiatric Institute & Clinic, ranked 10, 10, 1 respectively under efficiency and 7, 7, 8 under profitability when compared with other hospitals in Pennsylvania. The bottom ranks for Montefiore and Presbyterian under profitability and the low scores of all three hospitals under efficiency are indicative of the disadvantageous position of UPMC in the managed care environment.

DATA COLLECTION AND INTERPRETATION

Because the ANP gives the best results when a system is thoroughly represented, it is necessary to gather as much relevant data as possible so as to provide a realistic representation of the networks under observation. We determined that it was important to gather a large amount of data on hospital strategies from which we could select a few of the most realistic strategies for UPMC to adopt. From these strategies and our information sources, we then established the other clusters and elements that composed our networks.

To establish the interacting factors and clusters for the feedback networks, we reviewed a variety of articles from major medical journals including: Hospitals and Health Services Administration, Modern Health Care, Hospitals, Health Care Manage Review, Healthcare Financial Management, and Hospitals and Health Networks. We looked specifically for articles that pertained to university hospitals and strategic management. We were able to find articles that pertained to each of our strategies. From these articles, we identified the major factors that would both affect and be affected by the strategies. From these factors we identified those that were relevant to UPMC's case.

THE PROBLEM STRUCTURE

Our approach is to use 3 control models to optimize the alternatives with respect to benefits, costs and risks. In Benefits model we wanted to maximize the benefits to the UPMC, so in every pairwise comparison we needed to answering the question: Which element of the pair yields the greatest benefit?. In our Costs model we wanted to determine which factors have the highest costs or pains. In the Risks

model we evaluate which factors have the greatest risk. The three are then synthesized at the end of the analysis to attain a benefits/(costs x risks) ratio for the overall value of each strategy. This comprehensive approach is necessary to successfully model the complexity of the given problem. Below is a diagram of our overall model structure (see Figure 90) which contains three control models in the form of hierarchies: Benefits, Costs and Risks, each containing the control criteria of Social, Economic and Political. Thus there are nine control criteria in all, each with a network sub-model of its own.

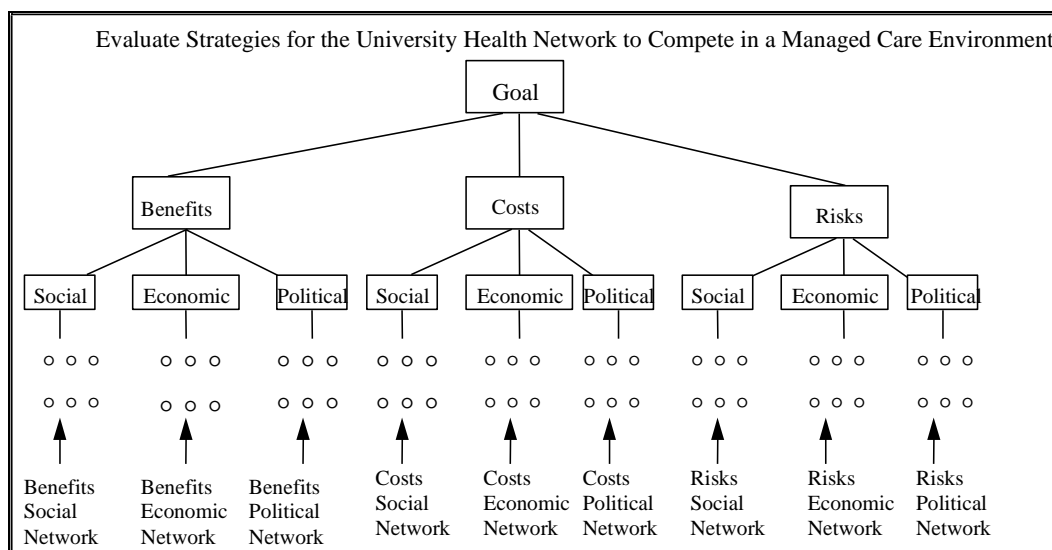


Figure 90. A Schematic of the Overall Model Structure

CONTROL CRITERIA AND NINE FEEDBACK NETWORKS

In the ANP a control hierarchy is necessary in order to make meaningful comparisons among network factors. The Economic, Political and Social control criteria provide a frame of reference within which the judgments are made. Each of the 3 models are evaluated under each of the 3 control criteria so that there are a total of nine feedback networks in which judgments must be made. These control criteria are general enough to encompass the wide range of factors influencing the success of the UPMC in a managed care environment. Under each control criterion, there is distinct supermatrix of impacts referred to as a network. To derive priorities for the control criteria, we pairwise compared them with respect to their importance under each of the sub-goals of benefits, costs, and risks. Based on the goal of successful competition in an increasingly managed healthcare environment, the economic factors were assumed to be the most important. The judgments are fairly consistent across the 3 models may be approximately summarized as follows for benefits, costs, and risks:

- the economic factors are very strongly more important than the social factors;
- the economic factors are strongly more important than the political factors;
- and the political factors are strongly more important than the social factors.

The priorities of the control criteria are shown for the three models in Table 29.

Table 29. Priorities for the Control Criteria in the Benefits, Costs, and Risks models.

	<i>Benefits</i>	<i>Costs</i>	<i>Risks</i>
Economic	0.6986	0.7417	0.7194
Political	0.2370	0.2185	0.1510
Social	0.0643	0.0668	0.0576

These weights are essential for computing the overall priorities of the strategies. The networks under the Economic criterion will have the most impact for all 3 models, followed by the Political networks, and the Social networks will contribute the least to the final priorities. In our opinion, to be more competitive and attract more clients, the UPMC will focus on the economic impacts of the alternative strategies. Although the social networks are modeled to show the extensive paths of social impact, the weights of these influences are less significant with respect to the goal being studied in this analysis. These priorities suggest that the overall rankings of strategies are influenced by the economic factors an average of 3.7 times more than the political factors and 11.5 times more than the social factors. Sensitivity of the results is explored at the level of the control criteria, by asking the question, "If our framework changes and the social and political factors gain in importance, how will the ranking of the alternatives change?".

NETWORKS

Each of the models has a network of clusters underneath the control criteria so that there are a total of nine networks (the 3 models times the 3 control criteria). The networks are similar in structure but are not identical. The connections between the clusters and their component elements are unique and express the important direct influences between elements. Indirect influences are captured by the overall flow of weight through the network. These unique connections and flow patterns give rise to what can be called a "profile" for each network. The clusters represented in the networks include: the important actors inside the UPMC, the alternative strategies, the consumers of healthcare, the variety of services, the quality of the UPMC functions, public relations, and internal stakeholders (see Table 30).

In the section below, we describe in detail the Economic Benefits, Costs and Risks networks and the results from each. Each section shows pictures of the cluster connections and the elements of the clusters, and explains the important connections and results for the given networks. Due to space limitations, the details of the analysis for the three Social and three Political networks are omitted and only the results are given.

Table 30. List of Clusters and Elements (Not all Clusters appear in every one of the 9 Subnetworks) .

<i>Cluster Names</i>	<i>Cluster Elements</i>
Clients	Businesses - employers who offer some type of health coverage plan to their employees. Consumers - individuals who purchase health coverage independently.
Competition	Insurers - companies that sell health insurance plans and pay for insured medical costs. Competitors - the other hospitals and medical centers, health maintenance organizations, and preferred provider organizations in Pittsburgh that directly compete with the UPMC.
Convenience	Time - general time involved in receiving care including time required for scheduling, travel to appointment, and wait times.
Internal Stakeholders	Safety - the safety of the location for healthcare. Physicians - the medical doctors working for the UPMC. Administrators - the planners, managers, and decision makers of the UPMC.
Public Relations	Alliances - organizations outside of the UPMC (insurers, hospitals, physicians networks, etc.) who have strategic alliances with some aspect of the UPMC. Staff - the non-physician, non-administrator sector of the UPMC employees.
Quality	Public Relations - actions taken to improve the UPMC's public image through television, newspaper, and radio advertisement. Specialty - the quality of the non-general health services. Diversity - the range of health services offered by the UPMC.
Strategies	Care - the quality of the general health services. Research - the quality of research performed at the UPMC. Improve and Measure Outcomes - measure how effective they are in delivering care and improving the outcomes to increase the attractiveness of the services.
Variety of Services	Capitation - negotiate insurance contracts to accept a fixed payment per patient per month. Develop a Primary Network - increase the number of primary care physicians affiliated with the UPMC to increase referrals and open primary care centers in the greater community. Internal Cost Reduction - general reduction in cost overhead through downsizing of facilities and employees and reduction of higher cost diagnostic procedures while maintaining or improving the current quality of care. Teach Primary Care - shift the focus of their teaching hospitals from curative specialty care to preventative primary care. Internal Medicine and Surgery - curative care including specialty services, procedures, and overnight hospitalization. Cancer Treatment - cancer treatment care. Outpatient Care - preventative care and shorter term medical treatment.

CLUSTERS, ELEMENTS, LINKS AND JUDGMENTS FOR THE ECONOMIC NETWORK MODELS

Clusters are collections of what are considered here to be relatively homogeneous elements. The elements are grouped based on some characteristics that they have in common such as being criteria that belong to a specific parent node in a hierarchy. Elements within each cluster are linked to elements in other clusters based on the influence between them. Clusters are linked to one another based on whether or not any elements within them are linked. If there are arrows in both directions between clusters, there is feedback influence occurring. The loop from one cluster to itself as shown by the white semi-circles indicates that there is influence between the elements within a specific cluster. The lines between the clusters in the macro view (shown below in Figure 91) indicate that some or all elements within an influenced cluster are compared for their importance with respect to some or all elements in the influencing cluster. The micro view with the elements showing is shown in Figure 92. If a cluster has a line with an arrow pointing to another cluster, it means that the latter influences the former. The pairwise comparisons for elements are performed using pairs of elements in the influencing clusters by asking, "What is the relative importance of the influencing elements on the element being influenced?" The pairwise comparison for clusters are performed by asking, "What is the relative importance of cluster A over cluster B with respect to the control criterion (in this case economic benefits)?"

The weights in the system are determined for the control criteria, clusters, and elements. First, each set of control criteria are weighted by pairwise comparison with respect to the overall goal of the model. Clusters are weighted by pairwise comparing every clusters impacts on every other cluster with respect to the control criterion in their network. Elements in each cluster are weighted by pairwise comparing them

with respect elements which they are linked within their own cluster (inner dependence) or between clusters (outer dependence). Weights are composed from the top down by multiplying the weights of the control criteria times the weights of the clusters times the weights of the elements. Table 30 gives a micro or expanded view of the clusters and their elements for the Economic Benefits network.

Once all of the weights have been determined for the control criteria clusters and elements in all of the networks, it is necessary to compute the benefits/(costs times risks) for the alternatives.

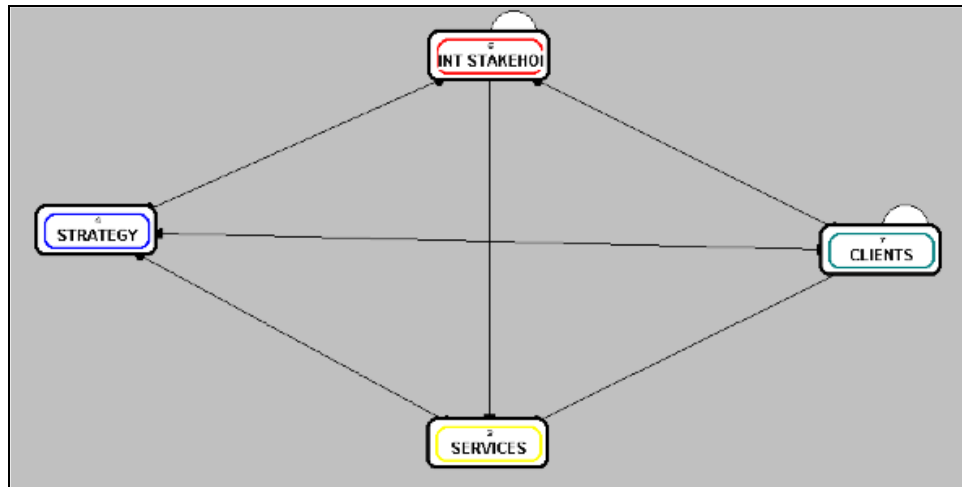


Figure 91. Macro View of the Clusters of the Economic Benefits Network.

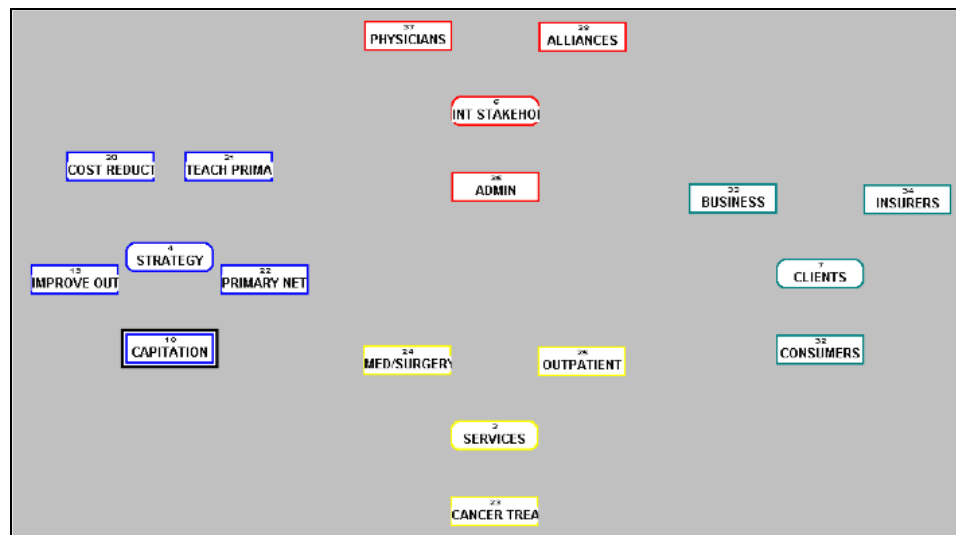


Figure 92. Micro View of Clusters and Their Elements for the Economic Benefits Network.

When viewing the Network from an economic perspective (economic perspectives adapted from interviews with Geary, DDI Consulting & Santana), strategies are influenced directly by internal stakeholders and insurers. An example of insurer influence to healthcare strategy formulation and economic benefits involves Desert Hospital in Riverside County. Hospital executives at Desert realized that they could be active in converting to managed care or wait until HMO's roll into town and tell them

what to do. Desert decided to developed a primary care network strategy in response to potential insurer (HMO) demands. (*Hospital Magazine*, 1991)

Indirectly, business and end consumers influence the selection of strategy through demands for more cost effective services offered through the insurers they contract with. Alternatively, Strategies influence internal stakeholders and the type and level of services provided. For instance the strategy of developing a primary care network will shift the focus of services away from highly specialized care to preventative care. (Hunt, Northwestern Healthcare Network, Chicago) Additionally, selecting and implementing a Primary Care Network strategy will increase the number of referrals to the medical center, in turn boosting revenues.

Our comparison of strategies within the Economic Benefits network determined that developing a Primary Care Network carried the most weight. There is no disputing that economic considerations are the major consideration to healthcare providers in formulating strategies. Developing a Primary Care Network strategy responds to business and insurer demands for Managed Care and potentially "locks in" revenue for the medical center.

ECONOMIC BENEFITS RESULTS

With respect to economic benefits, developing a Primary Care Network carried 34% of the local weight. Capitation came in second with 31% of the local weight. Successfully developing a primary care network will allow a healthcare provider the ability to pursue capitation arrangements with insurers (Hunt, Northwestern Healthcare Network, Chicago). Cost Reduction carried 18% of the local weight. Teaching Primary Care and Improving Measurement Outcomes both carried 8% local weights. Table 31 shows the local priorities for the Strategies under the Economic Benefits control criterion as well as the contribution to their global priorities.

Table 31. Economic Benefits Results

Strategy	Local Priority Under Economic Benefits Criterion (0.6987)	Economic Contribution to the Global Priority for the Benefits Model = (Local Priority * 0.6987)
Primary Network	0.3432	0.2398
Capitation	0.3081	0.2153
Cost Reduction	0.1826	0.1276
Teach Primary Care	0.0846	0.0591
Improve Outcomes	0.0816	0.0570

ECONOMIC COSTS CLUSTERS, LINKS, AND JUDGMENTS

The Economic Costs network is composed of the following clusters: Strategies, Internal Stakeholders, Services, and Clients. We did not include a Competition cluster because it is difficult to estimate how UPMC's competitors' decisions will affect the medical center in economic terms. We did not include a Quality Cluster since it is uncertain whether quality will be improved or harmed based on the strategy(s) selected. The links and level of influence between clusters and individual elements are similar to those described in the Economic Benefits network in terms of Clients, Internal Stakeholders, and Strategies. The macro view is shown in Figure 93 and the micro view is shown in Figure 94.

Our comparisons of the strategies determined that Capitation had the most weight in this Network. This result is consistent with the fact that insurance companies and the business community's demand for more cost effective health plans is putting pressure on hospital management to incorporate standardized procedures to monitor the service provided and the amounts of reimbursement insurers will provide. Capitation poses the biggest economic obstacle to UPMC as the stakeholders ponder how they will stay profitable in the face of strong insurer demands. The managed care environment forces healthcare providers to cut costs, otherwise sacrifice profits, by restricting the amount of reimbursement insurance companies will pay. (Self, May 1995)

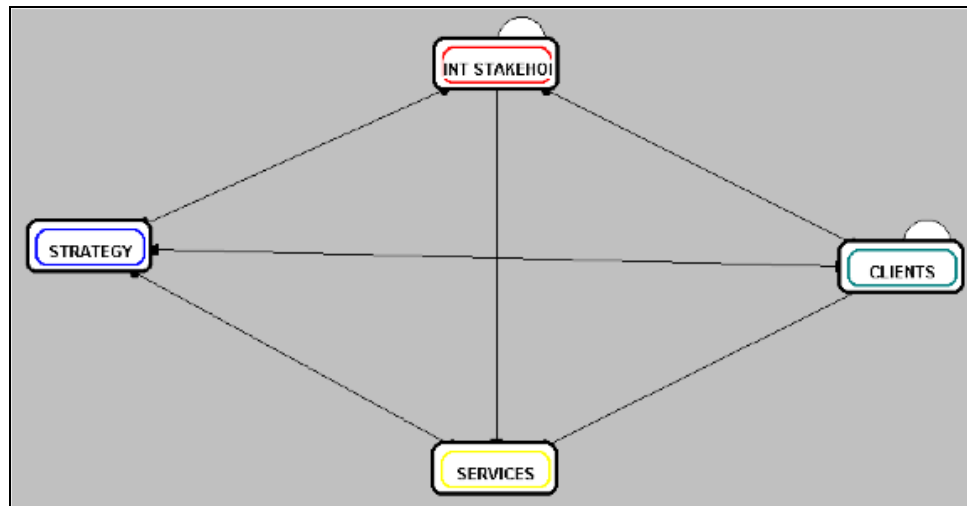


Figure 93. Macro View of the Clusters of the Economic Costs Network

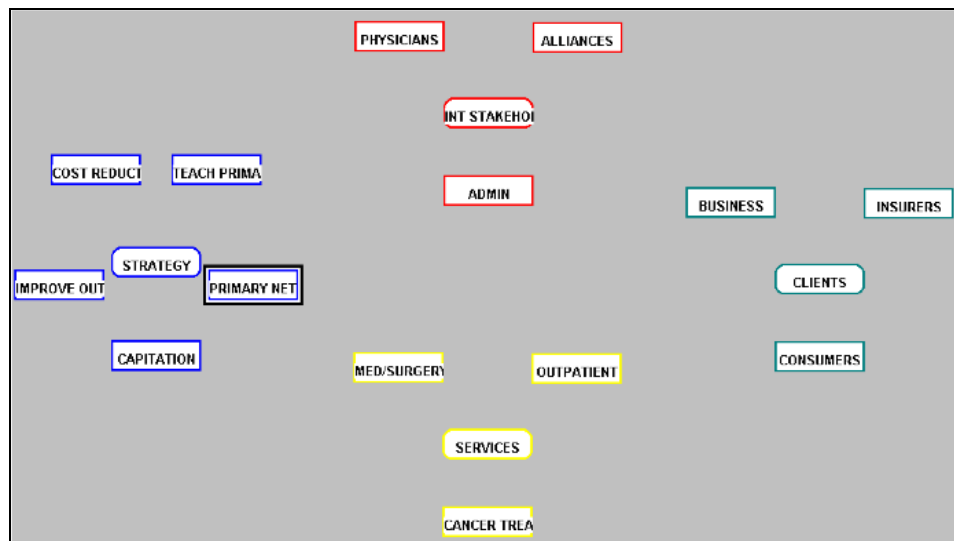


Figure 94. Micro View of Clusters and Their Elements for the Economic Costs Network.

ECONOMIC COSTS RESULTS

Table 32 shows the local priorities for the Strategies under the Economic Costs control criterion as well as the Economic contribution to their global priorities in the costs model.

Table 32. Economic Costs Results

Strategy	Local Priority Under	Economic Contribution to the
----------	----------------------	------------------------------

	Economic Costs	Control	Global Priority for the Costs Model = (Local Priority * 0.7417)
Primary Network	0.3119		0.2313
Capitation	0.3856		0.2860
Cost Reduction	0.0624		0.0462
Teach Primary Care	0.1107		0.0821
Improve Outcomes	0.1296		0.0961

With respect to Economic Costs, Capitation was found to carry 24% more weight than developing a Primary Care Network. Capitation and Primary Care Network carried local weights of 38.56% and 31.19% respectively. Improving Measurement Outcomes and Teaching Primary Care strategies carried local weights of 12.96 % and 11.07% respectively. The final strategy, Cost Reduction, carried a local weight of 6.24%. Cost reduction proved to be the lowest in relevance due to its internal focus as a strategy and not having as much impact from external influences.

ECONOMIC RISKS CLUSTERS, LINKS, AND JUDGMENTS

The Economic Risks network consists of six clusters: strategies, internal stakeholders, clients, competition, quality, and services. The strategy cluster is linked to the remaining five clusters, which in turn, link back to the strategy cluster. This network contains more clusters and links than the other two economic models because of the degree of uncertainty surrounding the effects of implementing the different strategies. The additional links in this network were: competition and quality of care delivered. The macro view is shown in Figure 95 and the micro view is shown in Figure 96.

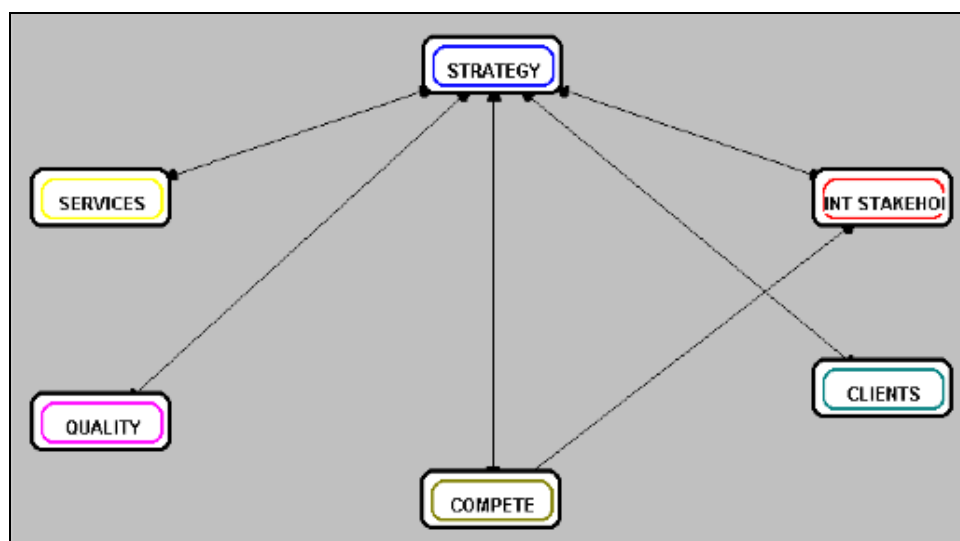


Figure 95. Macro View of the Cluster Connections of the Economic Risks Network.

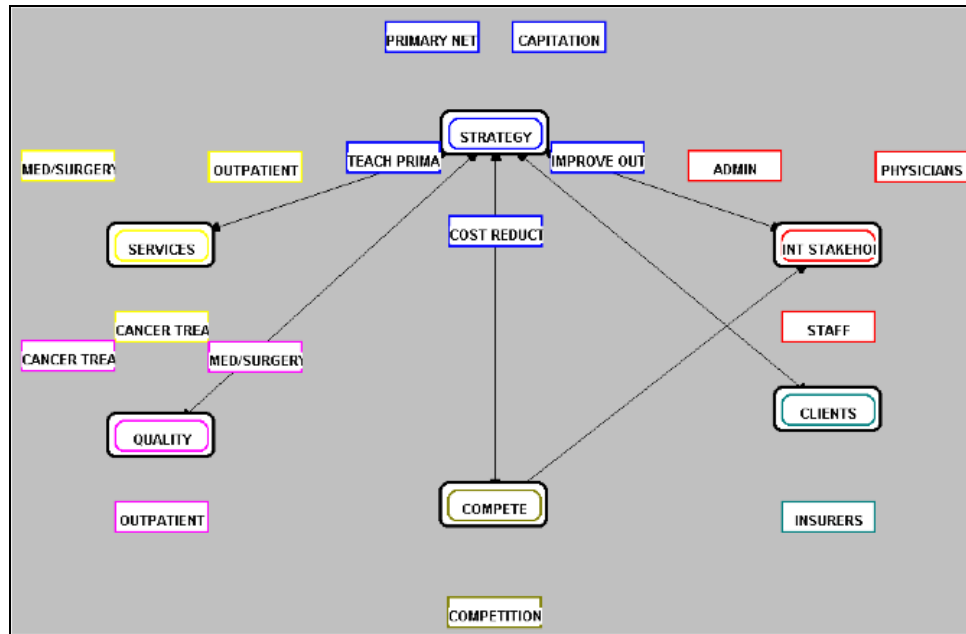


Figure 96. Micro View of Clusters and Their Elements for the Economic Risks Network

Since managed care is in the infant stages in Pittsburgh, UPMC and its competitors will be implementing various strategies at various points in time during the next few years. Due to the uncertainty of competitors' decisions, we believe that the competition cluster should be placed under the risks criterion and not under benefits or costs. We also placed quality of care under risks and not benefits or costs. Although UPMC must become more efficient in the managed care environment, it is uncertain whether they can maintain the same level of quality with fewer resources.

ECONOMIC RISKS RESULTS

Table 33 shows the local priorities for the Strategies under the Economic Risks control criterion as well as the contribution to their global priorities.

Table 33. Economic Risks Results.

Strategy	Local Priority Under Economic Risks Criterion (0.7194)	Economic Contribution to the Global Priority for the Risks Model = (Local Priority * 0.7194)
Primary Network	0.1161	0.0835
Capitation	0.4466	0.3213
Cost Reduction	0.2318	0.1668
Teach Primary Care	0.1157	0.0832
Improve Outcomes	0.0898	0.0646

The two largest risk local priority weights were: Capitation with 45% of the total priority and Cost Reduction with 23% of the total priority. Primary Network, Teaching Primary Care, and Improve Outcomes contained the following weights: 12%, 12%, and 8% respectively. Since the capitation concept has not yet been fully embraced by the Pittsburgh market, its impact is uncertain. It follows that this strategy has the highest weight as an economic risk. Although cost reduction will have economic benefits in the short run, it is uncertain whether UPMC can effectively compete with less resources. For

this reason, it follows that a cost reduction strategy would have the second highest weight as an economic risk.

ECONOMIC MODELS OVERALL RESULTS

Table 34 gives the three economic results for the strategies. We found that the strategy, Improve Outcomes, has a low level of benefits, but its costs and risks are so low that it becomes the best alternative when considering economic criteria.

Table 34. Overall Results for the Economic Benefits, Costs and Risks Networks.

Strategy	Economic benefits Priority	Economic costs Priority	Economic risks Priority	BCR Ratio for the Economic Criterion
Primary Network	0.2398	0.2313	0.0835	12.416
Capitation	0.2153	0.2860	0.3213	2.342
Cost Reduction	0.1276	0.0462	0.1668	16.558
Teach Primary Care	0.0591	0.0821	0.0832	8.652
Improve Outcomes	0.0570	0.0961	0.0646	9.182

The Political and Social network clusters, connections and priorities were determine in the same manner as described in detail for the Economic networks.

Table 35 gives the overall political results. Again, Improve Outcomes appears to come out the highest. It is closely followed by Teach Primary Care and Cost Reduction.

Table 35. Overall Political Results

Strategy	Political Benefits Priority	Political Costs Priority	Political Risks Priority	BCR Ratio for the Political Criterion
Primary Network	0.0713	0.0606	0.0564	20.861
Capitation	0.0307	0.0342	0.0135	66.493
Cost Reduction	0.1053	0.0770	0.0207	66.064
Teach Primary Care	0.0160	0.0238	0.0388	17.327
Improve Outcomes	0.0137	0.0231	0.0216	27.457

OVERALL SOCIAL RESULTS

Table 36 gives the overall results for the social network models. The best strategy relative to the social criterion is once again Improve Outcomes. It is closely followed by Primary Network. From the three Social networks, we determined that Improve Outcomes had the highest benefit and the lowest cost and risk. Capitation and Cost Reduction had low social benefits and high social costs and risks. The low priorities of Capitation and Cost Reduction are not surprising considering that these two strategies are clearly less oriented towards providing social benefits to the Clients or the Stakeholders.

Table 36. Overall Social Networks Results

Strategy	Social Risks Priority	Social Priority	Costs	Social Priority	Risks	BCR Ratio for the Social Criterion
Primary Network	0.0152	0.0006		0.0070		3619.1
Capitation	0.0055	0.0196		0.0166		16.904
Cost Reduction	0.0049	0.0300		0.0232		7.0402
Teach Primary Care	0.0100	0.0072		0.0059		235.40
Improve Outcomes	0.0287	0.0038		0.0049		1541.4

FINAL RESULTS

Table 37 gives the overall benefits, costs, and risk results as well as the B/(C*R) ratio. The three sets of derived priorities are combined by dividing the benefits by the costs and the risks for the alternative strategies to arrive at a single ratio number which expresses the overall utility of the strategies. This combination is meaningful because the derived priorities are ratio scales and the product and quotient of ratio scales is a ratio scale (Saaty, *Fundamentals of Decision Making*, p. 164).

We found that Improve Outcomes was the overall best alternative for UPMC to pursue. It is not extremely beneficial, but its costs and risks are low making it very attractive. Primary Network, Cost Reduction, and Teach Primary Care are all very close to one another in their priorities as strategies. Their priorities are similar because they overlap in the manner in which they are implemented. If UPMC is to pursue the Primary Network strategy, it must also reduce costs and teach primary care to its doctors and staff. Captivation was found to be an unattractive alternative at this point in time. Although its benefits are the second highest of the strategies, its costs and risks are very high which make it a strategy that should be pursued only if all else fails.

Table 37. Overall Benefits/(Costs*Risks) Results

Strategy	Benefits Priority	Global	Costs Priority	Global	Risks Priority	Global	Benefits/(Costs *Risks)
Primary Network	0		0		0		7.5939
Capitation	0		0		0		2.1062
Cost Reduction	0		0		0		7.3669
Teach Primary Care	0		0		0		5.8829
Improve Outcomes	0		00		0		8.8708

SENSITIVITY ANALYSIS

When we increased the priority of the Social control criterion in carrying out sensitivity analysis, the BCR ratio for the Improve Outcomes strategy became even more dominant due to its relatively strong social benefits and low social risks and costs (see Table 38 and Table 39). We performed a second sensitivity

analysis where the relative importance of the Economic criterion was decreased below the other two criteria. In this analysis, Improve Outcomes received a similar boost in its overall BCR ratio. It is interesting to note that the both the top and bottom ranked strategies maintained their rank across the three analysis scenarios. However, the middle three strategies (Primary Network, Cost Reduction, and Teach Primary Care) underwent rank reversal. The Primary Network strategy dropped from second place in the initial results to fourth in the two subsequent analyses; this is due its high Economic BCR ratio and low Political BCR ratio. The results of the sensitivity analyses suggest that UPMC ought to poll all of its top decision makers on the importance of the control criteria; this step would insure the results are consistent with their organizational values.

Table 38. Overall Priorities of the Control Criteria for Three Analysis Scenarios

Control Criteria	Overall Priorities	Priorities Adjusted for Increased Social Importance	Priorities Adjusted for Decreased Economic Importance
Economic	0.7309	0.5470	0.1094
Social	0.0639	0.3445	0.3090
Political	0.2053	0.1085	0.5815

Table 39. Benefits/(Costs*Risks) Ratios for the Three Analysis Scenarios

Strategy	Benefits/(Costs *Risks)	BCR for Increased Social Importance	BCR for Decreased Economic Importance
Improve Outcomes	8.8708	17.2075	15.8346
Primary Network	7.5939	6.5195	4.3378
Cost Reduction	7.3669	6.8327	5.2058
Teach Primary Care	5.8829	7.7599	4.9601
Capitation	2.1062	1.3324	2.4378

CONCLUSION

The results of our analysis indicate that the strategy with the highest utility for UPMC is to Improve Outcomes. Developing a Primary Care Network strategy ranked second. Our recommendation for implementation is that UPMC develop action steps to (1) establish outcome measurement processes throughout their provider services, and (2) formally develop a primary care network in southwestern Pennsylvania.

We realize that the two selected strategies require extensive research and development on the part of the Medical Center. The first strategy, Improving Outcomes, provides the potential for the University to measure itself along all three control criteria used in this analysis. For example, hospital administration and surgical procedure outcomes could be measured along budgetary and quality baselines. Publishing outcomes measurements and subsequent improvement of services and procedures can be used to notify the public, internal stakeholders, and clients of the progress UPMC is making in becoming more effective in providing healthcare.

Developing a primary care network is a formidable task. UPMC has already begun establishing satellite facilities outside its main medical centers. Therefore, infrastructure exists in the direction of networks. Additionally, UPMC's alliance with Blue Cross and Blue Shield provides support and expertise when it begins planning for a primary care system that focuses on preventative treatments. Blue Cross and Blue

Shield has already chosen UPMC as a preferred provider in its patient referral network. This fact alone, pushes UPMC further into competitive managed care position. (*Pittsburgh Post Gazette*, June 3 1995)

This team will provide a copy of our model and results to a UPMC administrator before the end of June, 1995. At this meeting we will discuss how this model and accompanying analysis can be used to advance UPMC's decision making process.

REFERENCES

ARTICLES

1. "Presbyterian University Hospital," The Best Hospitals in America, p.369-371.
2. 1994/1995 AHA Hospital Statistics, p116, 117, 158, 206, 240, 241
3. Pallarito, Karen. "Teaching hospital, med. school religion alliances," Modern Healthier, Feb. 6, 1995, p.36.
4. Pallarito, Karen. "Managed care may affect teaching hospitals' credit," Modern Healthier, Aug. 22, 1994, p.14.
5. Pallarito, Karen. "Hospital-medical school alliances shifting," Modern Healthier, Jul. 11, 1994, p.4.
6. Sandrick, Karen. "Primary care push-Teaching hospitals start programs to embrace MD base," Hospitals, Mar 20, 1993, p.56-57.
7. Mangan, Doreen. "Now look who's counting primary-care doctors," Medical Economics, Jul. 11, 1994, p.24-35.
8. Boosalis, Mary. "Tracking the best hospitals," Healthier Forum, Jan./Feb. 1993, p.53-57.
9. UPMC Department of Communication and PR, "University of Pittsburgh Medical Center".
10. "Financial Turnarounds: how two hospitals found profitability", Hospitals, Jan 20, 1990.
11. "Hospital excess switch hats--From provider to buyer of care", Hospitals, Feb. 20, 1990.
12. "The Quality March: Part Three of a National Survey of Quality Improvement Activities" Hospitals, Feb. 20, 1990.
13. "Welcome to the World of Managed Care", Self Magazine, March, 1995.
14. Leshner, Richard. "The Stakeholders-Leaders of Special-interest Groups Outline Their Ideas on Reform," Hospitals & Health Networks, Jan 20, 1994.
15. "Outcomes research: Hospitals Face Confidentiality Concerns", Hospitals, Jan 5, 1992.
16. "Primary Care: Hospitals Begin to Target Community Needs", Hospitals, April 5, 1990.
17. Henderson, John C. "Aligning Business and Information Technology Domains: Strategic Planning in Hospitals." Hospital & Health Derives Administration, Spring 1992.
18. "Proactive Executives: Prospering in Tough Times", Hospital, March 20, 1993.
19. "Restructuring Improves Hospital Competitiveness", Healthcare Financial Management, Nov 1992.

20. McManis, Gerald. "Putting Your Strategic Plan into Effect", Modern Healthcare, Jan 14, 1991.
21. Hoffman, Irene L. "Implementing the Medicare Electronic Remittance Advice", Healthcare Financial Management, Jan 1995.

BOOKS

1. Saaty, Thomas L. The Analytic Network Process, RWS Publications, 4922 Ellsworth Avenue, Pittsburgh, PA, 1996.
2. University of Pittsburgh Medical Center, Academic medical centers and reform, 1993.
3. Paull, Barbara. A Century of medical excellence: the history of the University of Pittsburgh School of Medicine, 1986.
4. University of Pittsburgh Medical Center, UPMC Forum, 1992.
5. Eastaugh, Steven R. Medical economics and health finance, Auburn House, 1981.
6. Smith, Howard L. Competitive Hospitals; management strategies, Aspen Publishers, 1986 .
7. Nackle, John G. Cost management for hospitals, Aspen Publishers, 1987.
8. Shouldice, Robert G. Introduction to managed care: health maintenance organizations, preferred provider organizations, and competitive medical plans, Information Resources Press, 1991.
9. Flood, Ann Barry. Hospital structure and performance, Johns Hopkins University Press, 1987.
10. Engineering a hospital turnaround, American Hospital Pub., 1993.

EXAMPLE 4. HONG KONG AND THE FINANCIAL CENTERS IN ASIA IN 2000

This paper was written by Gang Hu, Chia-Shuan Huang, Hong Li, Torsten Schmidt, and Yu-Chang Wang of the Katz Business School, 1997.

EXECUTIVE SUMMARY

The purpose of this study is to examine the potential impact on the financial center in Asia through the takeover of Hong Kong by the Chinese government in 1997. The interests of the two parties involved may not be the same, which may result in uncertainties in the future of Hong Kong as the Asian Financial Center. The analysis of this paper focuses on the following questions:

- What set of criteria does an Asian location have to meet in order to be a Financial Center?
- Which city is the best candidate for the Financial Center in the Asian-Pacific region in 1996?
- To the interest of the Chinese government, what will be the most likely Chinese policies towards Hong Kong after 1997?
- What impact will these Chinese policies impose over Hong Kong as a Financial Center?

- Which city is the most likely candidate for the Financial Center in the Asian-Pacific region in year 2000?

These five questions are studied with the methodology and technique provided by a combined AHP and ANP system. A dual-model approach has been developed. The first model, the “Financial Center model” which is an ANP model, is established to examine the first two of the questions above. The second one, the “Mainland China Policy model” which is an AHP model, is focused on the third question above in order to generate a policy package most likely to be adopted by the Chinese government. And based on changes in political, economic and social environments incurred by the estimated policy package, the “Financial Center model” has been reevaluated. The forth and fifth questions above are thus investigated.

Major results drawn from this study include:

- 1) The first result from the Financial Center model: If the Chinese government is able to maintain the current status of Hong Kong, Hong Kong would still be the Financial Center in the Asia-Pacific in 2000.
- 2) The second result: For the interests of the mainland Chinese government, no negative policy should be adopted towards Hong Kong after 1997. A careful and sensitive approach towards the future Hong Kong policy is suggested by this result, which is also reinforced by the next result.
- 3) The third result from the Financial Center model: Although Hong Kong may still be the best choice for the Financial Center, Singapore will become a strong competitor for the Center in 2000

1. INTRODUCTION

The future of Hong Kong has been a hot issue for a long time, especially at the current moment when less than one year is left before China reclaims it. After 140 years of British rule, Hong Kong has been transformed into a world city with a population of 6 million. However, another fundamental transformation will be embarked since the 1984 Sino-British agreement mandated the end of British colonialism and the sovereignty retrocession to the People’s Republic of China on July 1 1997.

The past two decades have seen China emerge from behind its Bamboo Curtain to assume center stage in Asia. Unfortunately, the characteristics of Asia’s new leading player is so complex that the only words which may seem to describe the country with any accuracy are contradictions, inconsistencies and anxieties. Uncertainties surrounding the shape of the post-1997 Hong Kong present such a typical puzzle attracting researchers to search for its solution.

Contradictory opinions on the 1997 transition can be seen in many publications. On the one hand, for example, it may be considered that while China’s development is creating insatiable demand for the territory’s goods and services, Hong Kong, with its huge investment commitments, stands to capitalize on its proximity to China as well as her strategic location in the region and after 1997 would continue to expand and diversify into one of the most important business and financial centers in the world. On the other hand, some people think that Hong Kong is no more fascinating to people outside Hong Kong, including the residents and investors alike. It means that the China factor has become a disturbing and destabilizing negative force underlining the imperfect transition and the crisis of confidence in Hong Kong.

Most concerns around the 1997 transition as such are politically- and socially-related and they actually can not be readily quantified under conventional frameworks. The ANP/AHP system provides useful tools and methodology needed to study the complex transition issues as a whole under a rigorous and comprehensive approach.

2. GOALS OF THE PROJECT

Identification of Major Issues in the Relationship between Mainland China and Hong Kong as a Financial Center:

China might be the next super power in 21st century, provided that it is able to maintain its economic growth. As a one of the most important gates to the world financial market, Hong Kong has provided for China tremendous economical opportunities and benefits in the past. One may safely say that “Hong Kong is to China what New York is to USA.” There is no doubt that keeping Hong Kong as a financial center of Asia could benefit China enormously. After China assumes the total control in 1997, however, will China still have enough sincerity to implement the approach of the “One country, two systems”, to keep Hong Kong as a financial center as prosperous after 1997?

In order to study questions like this, this study first identified the prerequisites for a financial center as follows which bring in the potential conflicts to the China’s typical communist policies:

The free inflow & outflow of capital and people

The free flow of information — a wide range of information including political information that affects financial markets

Western independent legal system

Could western freedom and communist totalitarianism with difference in those hot issues as such co-exist without major conflict? Which one has greater impact? If the former dominated, China might become a more democratic country in 21st century, which is the common hope in the outside world. If the latter, China might become the world’s new super villain and destroy Hong Kong at all costs, as it did to the students in Tienanman Square in 1989. China’s policy to Hong Kong is a good index to forecast its future intention to the Asian-Pacific region.

STUDY SUBJECTS

Due to the doubt on Hong Kong’s future, many Asian cities now are eager to replace Hong Kong as Asia-Pacific financial centers after 1997. Asia’s market is and will be the world’s fastest growing market in next century. A financial center could bring unimaginable benefits to the host country. This study is aimed at providing suggestions to major players in the financial industry concerning their strategic decisions on the location of their headquarters in the Asian-Pacific region after 1997. After comparing a set of subjective and objective factors, we believe that overall, Hong Kong, Singapore and Tokyo will be the appropriate candidates for Asia-Pacific business and financial centers after 1997. For these three cities, our research questions are focused as the following:

What set of criteria does an Asian location have to meet in order to be a financial center? In other words, what are the factors affecting the decision making about financial center most?

Which city is the best candidate for the financial center of Asia in 1996?

What will be the most likely policies of the Chinese government towards Hong Kong after 1997?

What impact will these policies have towards Hong Kong as a financial center?

Which city is the most likely candidate to be the financial center of Asia in year 2000?

TARGET GROUPS

The potential target groups of this study are listed as the following:

The Chinese government

The Singapore government

The Japanese government

The Hong Kong authorities

Present and potential financial corporations
Other consulting groups interested

STRUCTURE OF MODEL AND PROCESS OF MODEL CONSTRUCTION

The following are the assumptions of the model:

In order to reduce complexity of the study, the following assumptions have been made:

Rationality of the Chinese policies. The Chinese government policies would be made on the rational basis known to the public. No extraordinary change in the decision mode of the Chinese government would occur.

Sufficient political power and resources will be available to implement the rational policies as an entire package by the Chinese government.

A relative stable macro-political environment in Asia exists. Major players such as Singapore and Tokyo will still act strategically in a similar way in year 2000 as they do today.

The time frame is focused on five years: 1996 to 2000.

The representation of reality with limited number of the factors by this study is sufficient.

DESCRIPTION OF THE FINANCIAL CENTER MODEL (MODEL 1)

The following steps are taken to construct the first model (the Financial Center model):

The establishment of the goal to be studied: Which city could be the Asia-Pacific's financial center in 1996?

The establishment of the control hierarchy: The following two levels of logical structure which affects decision-making of multinational enterprises are listed:

The types and the associated degrees of possible benefits, costs, opportunities and risks linked with a city under examination: Four categories, benefits, costs, opportunities and risks, are developed to evaluate the related positive and negative effects that a city could impose to the financial business concerned.

The economic, social, and political factors under each category (see section 3.5.) are listed as the possible factors under each category, and the relative advantage or disadvantage of each two cities are evaluated under each factor; then the relative importance of each factor to each sub-criteria is compared, and thus the relative importance of each criteria to a financial center is measured.

DESCRIPTION OF THE CHINA GOVERNMENT MODEL (MODEL 2)

Because Hong Kong will be taken over by China after 1997, China's policies towards Hong Kong has absolute affect on whether Hong Kong could still be the Asia-Pacific's financial center after 1997. Hence, we have selected 18 factors, derived from the results of the first model, which prove to have major affects on the current influence from China over Hong Kong. We have designed three situations, purely positive represented by "+", purely negative represented by "-", and a mixed type in between represented by "0". For example, "free information+" means that China will allow free information in Hong Kong the same as before 1997, and the cost of such policy is 15.514 (please see Appendix). Thus, +, -, and 0 are identifiers for the consequences of the implemented Chinese policies towards Hong Kong as a Financial Center. We use three ratios to support our judgment in the first model.

SPECIAL FEATURES OF THE APPROACH

a). Special concerns for the analysis:

Due to the uncertainties involved in the future Chinese policies mentioned at the beginning of the paper, two major considerations arose in the analysis:

How to include the impact of the mainland China government's policies after 1997 into our financial center model?

How to analyze and to estimate the consequences of the future Chinese policies which will have strong influence over Hong Kong after 1997?

b). A dual-model approach provides complementary structures for the two models in order to answer these two questions:

The Financial Center model provides the relevant factors for a focused examination under the China model in order to find the relevant factors which may be changed by the Chinese government;

The China model provides a package of feasible (for the mainland China government only) and most likely policies to be adopted by the Chinese government after 1997. Based on this package of policies, the second evaluation of the Financial center model is made in order to estimate the future status of Hong Kong as a financial center by taking the consequences of the Chinese policies into account.

c). The built-in different perspectives for the two models:

The establishment of factors and the pairwise comparisons for the Financial Center model is only based on the requirements of a Financial Center in the Asian-Pacific region;

The establishment of the pairwise comparisons of policies for the mainland China government model is only based on the interests of the mainland Chinese government.

INFLUENCING FACTORS

Under our control hierarchy, we inserted the various factors which we have evaluated cautiously. These factors are classified under political, economic, and social categories and each of the three items is related to risks, opportunities, costs, and, benefits respectively. These factors are as follows:

A). Economic-Benefits:

1. Geographic advantage.
2. Free flow of information.
3. Free flow of people.
4. Free flow of capital.
5. Internationalization.
6. Investment.
7. Educated workforce.
8. Convertible currencies.
9. Assistance from government.
11. Modern infrastructure.
12. Deregulated market.

B). Political-Benefits:

1. Efficient government.
2. Independent legal system.
3. Assistance from government.
4. Free flow of people.
5. Free flow of information.

C). Social-Benefits:

1. Free flow of people.
2. Free flow of information.
3. Educated workforce.
4. Open culture.
5. Internationalized language.
6. Availability of business professionals.

D).Economic-Costs:

1. Labor cost.
2. Corruption.
3. Protection from government.
4. Operating cost.
5. Tax.

E).Political-Costs:

1. Tax.
2. Corruption.
3. Protection from government.

F).Social-Costs:

1. Environment.
2. Corruption.
3. Protection from government.

G).Economic-Opportunities:

1. Investment.
2. Access to potential market.
3. Regional economic growth, membership of international organizations (GATT, WTO).

H).Political- Opportunities:

1. Political credit.
2. Investment.
3. Membership of international organization (GATT,WTO).

I).Social-Opportunities:

1. Social wealth.
2. Access to potential market.

J).Political-Risks:

1. Political instability.
2. Instability of local government.
3. Political restriction.

K).Economic-Risks:

1. Instability of local financial market.
2. Inflation.
3. Competition from local business.

L).Social-Risks:

1. Industry resistance.
2. Public industry.
3. Instability of local society.

3. DESCRIPTION AND INTERPRETATION OF THE RESULTS

THE FIRST OUTPUT FROM THE FINANCIAL CENTER MODEL

First, we assessed the situation of all three cities in 1996. The outcome from this assessment would provide the frame of reference for the second output of the Financial Center model. In addition, it also validated overall Financial Center Model. After synthesis of the model, we got the results below:

Benefits

	Economic	Political	Social
Hong Kong	0.4131	0.5034	0.4416
Singapore	0.2836	0.1503	0.2164
Tokyo	0.3033	0.3465	0.342

Costs

	Economic	Political	Social
Hong Kong	0.2393	0.1922	0.2519
Singapore	0.2804	0.3625	0.1803
Tokyo	0.4803	0.4453	0.5678

Opportunities

	Economic	Political	Social
Hong Kong	0.4096	0.3813	0.4511
Singapore	0.2874	0.2935	0.2227
Tokyo	0.303	0.3252	0.3261

Risks

	Economic	Political	Social
Hong Kong	0.3086	0.4278	0.4387
Singapore	0.2186	0.4387	0.2365
Tokyo	0.4728	0.1335	0.3248

The overall result is listed below:

Alternatives	Rank	$(B*O)/(C*R)$
Hong Kong	1	1.6498
Singapore	2	0.906
Tokyo	3	0.5338

It is clear that Hong Kong has the highest ratio, 1.6498, which indicates that Hong Kong currently enjoys its solid position as the number one Financial Center in the Asian-Pacific region. This outcome is consistent with the general observations which suggest the validity of the modeling approach.

THE OUTPUT FROM THE CHINESE POLICIES MODEL

We selected 18 different factors in the Financial Center model which are highly dependent on the policies of the mainland China government. For each of the factors, we designed three situations (positive +, mixed 0, negative -), which denote the different outcomes from different Chinese policies under each factor. And then, in order to obtain the possible policies, we construct them into the Chinese government model, which is an absolute hierarchy model, including four sub-models: benefits, costs, opportunities and risks. After synthesis of the model, we derived the overall scores for each of the policies. Based on the scores, the most likely policies package shown below was developed.

The Package of Chinese Policies:

	Optimal and most likely policies
1	free flow of information 0
2	free flow of people 0
3	educated workforce +
4	convertible currency +
5	deregulated market 0
6	assistance from government +
7	inflation +
8	independent legal system 0
9	political restrictions 0
10	instability of local society +
11	availability of business professionals +
12	public insecurity +
13	corruption +
14	tax +
15	protectionist barrier 0
16	investment +
17	political credit +
18	access to potential market +

Among the 18 factors, as suggested by the results, the Chinese government tends to adopt positive policies on 12 of them and mixed policies on 6 of them (as shown in the table above).

THE SECOND OUTPUT FROM THE FINANCIAL CENTER MODEL

Based on the most likely policies package, we adjusted the input to the Financial Center model and ran the model the second time. This is, with the assumptions made in section 3.1, another evaluation of the three possible locations of the future financial center after 1997 in the Asian-Pacific region. The results are listed below:

Benefits

	Economic	Political	Social
Hong Kong	0.3814	0.4435	0.4238
Singapore	0.2992	0.2187	0.2306
Tokyo	0.3194	0.3378	0.3456

Costs

	Economic	Political	Social
Hong Kong	0.2966	0.2814	0.3045
Singapore	0.2561	0.3161	0.1657
Tokyo	0.4475	0.4025	0.5298

Opportunities

	Economic	Political	Social
Hong Kong	0.4096	0.3813	0.4511
Singapore	0.2874	0.2935	0.2227
Tokyo	0.303	0.3252	0.3261

Risks

	Economic	Political	Social
Hong Kong	0.4278	0.4278	0.4387
Singapore	0.4387	0.4387	0.2365
Tokyo	0.1335	0.1335	0.3248

The overall result is below:

Alternatives	Rank	$(B*O)/(C*R)$
Hong Kong	1	1.1822
Singapore	2	1.1093
Tokyo	3	0.5949

We can see that Hong Kong still maintains its status as the number 1 financial center after 1997 with the ratio 1.1822, but the gap between Hong Kong and other cities becomes much smaller. Especially, the position of Singapore (ratio 1.1093) becomes very competitive. The position of Tokyo improved only marginally.

LIMITATIONS OF OUR MODEL

Each factor in the Chinese policy model is considered separately

Due to the limited time and resources available, we could not research the cross-link effects between the factors in the Chinese policy model. When we construct the model, we focused on analyzing the future prospect of Hong Kong.

No inside knowledge from the Chinese government officials was available.

We adopted information available from public resources such as international or domestic journals, publications, and some official data from other countries.

The entire approach is based on the most-likely case scenario.

This is due to the type of estimation which is based on the most feasible policy package developed by the China model. But in “political” reality, this type of political package is seldomly implemented completely. The political decision makers may deviate from their designated policies for various political reasons. Additionally, no complex and open system can be understood and controlled completely. Therefore, deviations from the designed policy package are likely.

This model is structured based upon a time frame of five years.

For a longer time horizon, this model may provide an estimation with low external validity. The long-term study will require more detailed and concrete studies to analyze possible scenarios in a much larger scopes.

4. CONCLUSIONS

Based on the first output of the Financial Center model, it can be seen that if the Chinese government is able to adopt all the policies to maintain or even improve the current status of Hong Kong, it is quite likely that Hong Kong will remain as one of the important financial centers in the Asia-Pacific region.

Among the influencing factors on the financial center status, the Financial model reveals that 18 of these factors are dependent directly or indirectly on the government’s policies. Therefore, Hong Kong’s future as a financial center is highly dependent on the political attitude of the Chinese government.

Based on the result of our mainland China policy model, we found, among the 18 factors, the Chinese government should adopt positive policies on 12 of them, and mixed policies on 6 of them. In other

words, for the interests of mainland China (note not for the interests of Hong Kong), China should avoid implementing *negative policies*, as defined early in this study, towards Hong Kong as a financial center.

Based on the second output of the Financial Center model, Hong Kong will still maintain the financial status after 1997. But at the same time, Singapore will become very competitive. Therefore our conclusion is that if Chinese government adopts rational policies toward Hong Kong as estimated in this study, Hong Kong will still be the number 1 financial center of the Asian-Pacific region. But at the same time, the position of Hong Kong as a financial center will be weakened. If any more negative policies are implemented, Singapore will become the number 1 financial center of Asia-Pacific region followed by Hong Kong.

References for the Hong Kong paper

- Asia Study Ltd.. Key Trends and Statistics (China, Hong Kong, Singapore, Japan). 1996.
- Bowring, Philip. Decades-old Political Ties Speak Louder Than Money. South China Morning Post, August 12, 1996
- Broadfoot, Robert. China's Growing Impact on Asia. Political and Economic Risk Consultancy, Ltd. - PERC.. November 1995.
- Expert Choice, Inc. Decision Making with Dependence and Feedback: The Analytic Network Process (ANP) and ECNet Software, Guide, Manual and Examples. Expert Choice, Inc., Pittsburgh, 1996.
- Farh, Jiing-lih, Leung Kwok and David K. Tse. Managing Human Resources in Hong Kong: 1997 and Beyond. Columbia Journal of World Business, Summer 1995 v30 n2 p52 (8).
- Gordian, Gaeta. Hong Kong's Financial Industry in Transition: an External Perspective. Columbia Journal of World Business, Summer 1995 v30 n2 p42 (8).
- Kraar, Louis. The Death of Hong Kong. Fortune, June 26, 1995 v131 n12 p118 (10).
- Lao, Francesco and Xi Sisci. China Lays Down Law for Stability, Asia Times, October 11, 1996.
- Lee, Martin Q. C.. Business and The Rule of Law in Hong Kong. Columbia Journal of World Business, Summer 1995 v30 n2 p28 (5).
- Li, David K.P.. Enter the Dragon: Hong Kong's Growing Role in World Finance. Columbia Journal of World Business, Summer 1995 v30 n2 p34 (6).
- Pisani, Elizabeth. Feisty Tsang Challenges Singapore. Asia Times, October 9, 1996.
- Political and Economic Risk Consultancy, Ltd. - PERC. Country Risk Report: Hong Kong. January 20, 1997.
- Saaty, T. L. and Luis G. Vargas. Decision Making in Economic, Political, Social and Technological Environments: The Analytic Hierarchy Process. RWS Publications, 1994.
- Saaty, T. L.. Decision Making with Dependence and Feedback: The Analytic Network Process (ANP). RWS Publications, 1996.
- Saaty, T. L.. Fundamentals of Decision Making and Priority Theory with the Analytic Hierarchy Process. RWS Publications, 1994.
- Shirreff, David. A Hub to Replace Hong Kong?. Euromoney, February 1995, p80-82.
- Shirreff, David. Richard Hu's Rearguard Action. Euromoney, February 1995, p84-87.

References to Papers on Predicting and Forecasting with the AHP and the ANP

- Blair, A., R. Nachtmann, J. Olson, and T. Saaty. "Forecasting Foreign Exchange Rates: An Expert Judgment Approach," *Socio-Economic Planning Sciences* 21, 6 (1987), pp. 363-369.
- Blair, A., R. Nachtmann, and T.L. Saaty, 1993, "Incorporating Expert Judgment in Economic Forecasts: The Case of the U.S. Economy in 1992", in *Fundamentals of Decision Making and Priority Theory with The Analytic Hierarchy Process*, Saaty, Thomas L., 1994 RWS Publications, Pittsburgh, PA.

Gholamnezhad, H. "1995: The Turning Point in Oil Prices," *The Global Economy: Today, Tomorrow, and the Transition*. Edited by H.F. Didsbury, Jr., World Future Society, Washington, DC, 1985.

Gholam-Nezhad, H. "Oil Price Scenarios: 1989 and 1995," *Strategic Planning and Energy Management* 7, 1 (1987), pp. 19-31.

Saaty, T.L. and J.P. Bennett. "A Theory of Analytical Hierarchies Applied to Political Candidacy", *Behavioral Science* 22 (1977), pp. 237-245.

Saaty, T.L. and R.S. Mariano. "Rationing Energy to Industries: Priorities and Input-Output Dependence," *Energy Systems and Policy* 8, (January 1979), pp. 85-111.

Saaty, T.L., P.C. Rogers, and R. Pell. "Portfolio Selection Through Hierarchies," *Journal of Portfolio Management* 3 (1980), pp. 16-21.

Saaty, T.L. and L.G. Vargas. "Hierarchical Analysis of Behavior in competition: Prediction in Chess," *Behavioral Science* 25 (1980), pp.180-191.

Saaty, T.L. and H. Gholamnezhad. "Oil Prices: 1985 and 1990," *Energy Systems and Policy* 5, 4 (1981) pp. 303-318.

Saaty, T.L. and M. Wong. "The Average Family Size in Rural India," *Journal of Mathematical Sociology* 9 (1983), pp.181-209.

Saaty, T.L. and L.G. Vargas. "Modeling Behavior in Competition: The Analytic Hierarchy Process," *Applied Mathematics and Computation* 16 (1987), pp. 49-92.

Saaty, T.L., D.S. Turner, "Prediction of the 1996 Super Bowl: An Application of the AHP with Feedback (the Supermatrix Approach)" Proceedings of the 4th International Symposium on the Analytic Hierarchy Process, Vancouver 1996, Expert Choice Inc., 5001 Baum Boulevard, Pittsburgh, PA 15213.

